OPTIONS AND MATHEMATICS (CTH[$mve095$] , GU[$MMA700$])
ASSIGNMENTS 2010

Please hand in solutions of at most 5 out of the following 6 problems at the latest Friday, May 14 at 13^{45}. Each problem gives at most $0.5/5 = 0.1$ credit points on the final examination in May 2010.

You need a computer to solve Exercises 10 and 12 (b), (c). The MATLAB figures in Problems 8(b), 9, and 11 must be simple to understand and with defined label and axes. No parts of the MATLAB figures are allowed to be written by hand. Good luck!

Problem 7 A random variable X has the density function $f(x) = \frac{3x^2}{2} 1_{[-1,1]}(x)$, $x \in \mathbf{R}$. Find the characteristic function $c_X(\xi) = E\left[e^{i\xi X}\right]$, $\xi \in \mathbf{R}$.

Problem 8 (MATLAB; title('Problem 8'), legend('Lognormal density of a stock price')) Consider the geometric Brownian motion model of a stock price process $(S(t))_{t\geq 0}$, where

$$S(t) = S(0)e^{(\mu-\frac{\sigma^2}{2})t+\sigma W(t)}.$$

(a) {Handwritten solution} Suppose $t > 0$. Find the density function f of the random variable $S(t)$. (b) Suppose $t = 1$, $S(0) = 1$, and $\mu = 0.05$. Plot the graph of the density function f for the values $\sigma = 0.3$ and $\sigma = 0.5$.

Problem 9 (MATLAB; title('Problem 9'), legend('50 log-Brownian asset prices')) Consider the geometric Brownian motion model of a stock price process $(S(t))_{0\leq t\leq T}$, where

$$S(t) = S(0)e^{(\mu-\frac{\sigma^2}{2})t+\sigma W(t)}$$

Plot 50 realizations of the stock price process for $S(0) = 1$, $\mu = 0.05$, $\sigma = 0.5$, and $T = 1$.

Problem 10 Let c, $cdelta$, and p be, respectively, the European call price, the European call delta, and the European put price. Suppose the input arguments are $\tau = T - t$, $S(0)$, K, r, and σ. Check the values of c against the call prices in "Introduction to the Black-Scholes Theory", (Version: 2010), page 90. (a) Complete with values of p (b) Complete with values of $cdelta$. (Hint: Use MATLAB)

Problem 11 (MATLAB; title('Problem 11'), legend('Theoretic value','Actual value') Consider a stock price process $(S(t))_{0\leq t\leq T}$, where

$$S(t) = S(0)e^{(\mu - \frac{\sigma^2}{2})t + \sigma W(t)}.$$

Suppose $h = T/N$ and $t_n = nh$, $n = 0, ..., N$. Define $cash(t_0)$ by the equation

$$c(t_0, S(t_0), K, T) = \Delta_C(t_0)S(t_0) + cash(t_0)$$

where $\Delta_C(t) = c_s'(t, S(t), K, T)$. Set

$$\Pi_1 = \Delta_C(t_0)S(t_1) + cash(t_0)e^{rh}$$

where r is the interest rate. Next define $cash(t_1)$ by the equation

$$\Pi_1 = \Delta_C(t_1)S(t_1) + cash(t_1)$$

and

$$\Pi_2 = \Delta_C(t_1)S(t_2) + cash(t_1)e^{rh}$$

and continue the process until maturity T. Plot the theoretic call prices $c(t_i, S(t_i), K, T)$, $i = 1, ..., N$, and the portfolio values Π_i, $i = 1, ..., N$ in the same figure when $S(t_0) = 100$, $K = 105$, $T = 0.5$, $\mu = 0.05$, $r = 0.03$, $\sigma = 0.35$, and $N = 130$. Finally, find $\Pi_N - (S(T) - K)^+$.

Problem 12 Suppose $f(x) = \sin(x+1)$, $x \in \mathbf{R}$, and let $G_1, ..., G_n \in N(0,1)$ be independent.

(a) Find the value of the integral

$$I = \int_{-\infty}^{\infty} f(x)e^{-\frac{x^2}{2}} \frac{dx}{\sqrt{2\pi}}.$$

(b) Find approximations of I using the estimates

$$MC_1 = \frac{1}{n}\sum_{k=1}^{n} f(G_k)$$

for $n = 100, 1000, 10000$, and 100000. (Hint: Use MATLAB)

(c) Find approximations of I using the estimates

$$MC_2 = \frac{1}{2n}\sum_{k=1}^{n}(f(G_k) + f(-G_k))$$

for $n = 100, 1000, 10000$, and 100000. (Hint: Use MATLAB)