

OPTIONS AND MATHEMATICS (7.5 hec)**(CTH[mve095], GU[MMA700])**<http://www.math.chalmers.se/Math/Grundutb/CTH/mve095/><http://www.math.chalmers.se/Math/Grundutb/GU/MMA700/>**Period 4, spring 2010 (50 hours)**

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Examiner: Christer Borell

Lectures (Borell): Week 11: Tuesday 10-11⁴⁵ (Euler); Thursday 13¹⁵-15 (KA). For the remaining lectures, see the Schedule at the website

[http://www.chalmers.se/math/SV/utbildning/
grundutbildning/kurser/fristaende-kurser/mma700](http://www.chalmers.se/math/SV/utbildning/grundutbildning/kurser/fristaende-kurser/mma700)

Exercises: Week 11: Friday 13¹⁵-15 (MVF31, Borell (English)) and (MVH12, Raufi (Swedish)). For the remaining exercises, see the Schedule at the website

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Textbook: Christer Borell, Introduction to the Black-Scholes Theory, Version: 2010 (can be purchased at the DC, Maskingränd, Chalmers)

CONTENTS**Week 11**

Financial derivatives of European and American types. Forward contracts. The Dominance principle. Put-call parity. Convexity properties of European call and put prices.

Exercises: Chapter 1.1: 1, 3, 4, 5, 6, 7, 8, 9

Week 12

Introduction to the binomial model. Arbitrage portfolio. Replicating and self-financing strategies.

Exercises: Chapter 2.1: 1, 2, 4; Chapter 2.2: 1, 3, 4, 5; Chapter 2.3: 3 (give an alternative solution)

Week 15

Basic concepts in probability: event, random variable, Markov's inequality, characteristic function, Gaussian random variables, independence.

Exercises: Chapter 3.1: 1, 2, 3, 4, 5, 6, 7, 8

Week 16

Random walk. Law of Large Numbers. Monte Carlo simulation. Central Limit Theorem.

Exercises: Chapter 3.1: 9, 10, 11, 12, 13, 14, 15, 17

Week 17

Brownian motion. The geometric Brownian motion model of a stock price. Some remarks on portfolio theory. Heat conduction, simple random walk, and Brownian motion. Probabilistic representations of solutions to the heat equation and some other parabolic differential equations. Simple random walk and the heat equation.

Exercises: Chapter 3.2: 3; Chapter 4.1: 1; Chapter 4.2: 1; Chapter 4.3: 1, 2, 3; Chapter 3.1: 16, 20

Week 18

The Black-Scholes model and differential equation. Call prices. European and American put prices. Simple currency derivatives. Options on forward contracts.

Exercises: Chapter 5.1: 1, 2; Chapter 5.2: 1, 2, 3, 4, 5; Chapter 3.1: 19

Week 19

More on the Black-Scholes model. Greeks and sensitivity analysis. The Black-Scholes prices of path-dependent options. Implied volatility.

Exercises: Chapter 5.3: 1, 2, 3, 4, 5; Chapter 5.4: 2; Chapter 5.5: 1; Chapter 5.6: 2

Week 20

Bivariate Brownian motion (Chapter 6, first 3 pages). A mean-variance approach to portfolio selection.

Exercises: Chapter 6.1: 1, 3; Chapter 8.2: 1, 2, 3; Chapter 8.4: 1, 2 (add: if $|\sigma_i| = 1$); Chapter 5.4: 3

EXAMINATION

Assignments handed in to the examiner

A number of exercises solved and handed in by the student at the latest Friday April 16 at 15⁰⁰ (Assignment 1) and Friday May 14 at 15⁰⁰ (Assignment 2) will result in a maximum of 0.5+0.5=1 point at the final examination in May 2010.

Written final examination (4 hours)

May 24, 2010, morning, v

August 28, 2010, morning

January 15, 2011, morning, v

Aid not permitted.

The test comprises 15 points (in May 2010 (plus the credit from the assignments) and to pass at least 6 points are required (at GU 11 points or more is graded VG; at Chalmers a result greater than or equal to 9 points and smaller than 12 points is graded 4 and a result greater than or equal to 12 points is graded 5).

At least 6 points are of theoretic nature and at least 3 of these are chosen from the following list:

Theorem 1.1.2; Theorem 1.1.3; Theorem 1.1.4; Theorem 2.1.1; Theorem 2.2.1; Theorem 3.3.1; Theorem 4.1.1; Theorem 4.2.1; Theorem 4.3.1; Theorem 4.3.2; Theorem 5.1.1; Theorem 5.2.1; Theorem 5.3.1 (only the formula for delta); Theorem 6.1.1.

Göteborg December 29, 2010
Christer Borell