

OPTIONS AND MATHEMATICS (CTH[mve095], GU[MMA700])
ASSIGNMENTS 2012

(must be handed in at the latest Friday, March 30, 2012 at 15⁰⁰)

1. Suppose $K > 0$. Find a portfolio consisting of European calls and puts with termination date T such that the value of the portfolio at time T equals

$$Y = \max(0, |S(T) - K| - \frac{1}{2}K).$$

2. (Binomial model with $T = 3$ and $d < r < u$) A financial derivative of European type has the payoff

$$Y = \begin{cases} X_2 & \text{if } X_1 = X_2 = X_3, \\ 0 & \text{otherwise,} \end{cases}$$

at time of maturity T . (a) Find $\Pi_Y(0)$. (b) The portfolio strategy h replicates Y . Find $h(0) = (h_S(0), h_B(0))$.

3. Suppose $D = \{(x, y); 0 < x < 1 \text{ and } y > 0\}$ and let (X, Y) be a random vector in the plane with the density function $f(x, y) = 1_D(x, y)(3x^2 + y)e^{-y}/2$. For which real t is the variance $\text{Var}(X - tY)$ minimal?

4. Suppose (X, Y) is a centred Gaussian random vector in the plane. Show that

$$E[X^2Y^2] \geq E[X^2]E[Y^2].$$