

OPTIONS AND MATHEMATICS (7.5 hec)**(CTH[mve095], GU[MMA700])**<http://www.math.chalmers.se/Math/Grundutb/CTH/mve095/1112/>**Period 4, spring 2012 (50 hours)****Teachers:** Christer Borell, Peter Helgesson**Examiner:** Christer Borell**Lectures** (Pascal: Borell): Week 11: Monday 13¹⁵-15; Wednesday 8-9⁴⁵; Thursday 13¹⁵-15. For the remaining lectures, see the schedule at the website[http://www.chalmers.se/math/SV/utbildning/
grundutbildning/kurser/fristaende-kurser/mma700](http://www.chalmers.se/math/SV/utbildning/grundutbildning/kurser/fristaende-kurser/mma700)**Exercises:** Friday 13¹⁵-15 (Pascal: Borell (Swedish)); (MVF26: Helgesson (English))**Textbook:** Christer Borell, Introduction to the Black-Scholes Theory, Version: 2011 (see the homepage of the course)**CONTENTS****Week 11, 8 hours**

Financial derivatives of European and American types. Forward contracts. The Dominance principle. Put-call parity. Convexity properties of European call and put prices. Introduction to the binomial model.

Exercises: Chapter 1.1: 1, 3, 4, 5, 6, 7, 8, 9

Week 12, 8 hours

The multi-period binomial model. Arbitrage portfolio. Replicating and self-financing strategies. Basic concepts in probability: event, random variable, Markov's inequality, characteristic function. Gaussian stochastic process.

Exercises: Chapter 2.1: 1, 4; Chapter 2.2: 1, 3, 4, 5, 6; Chapter 2.3: 3 (give an alternative solution)

Week 13, 6 hours

Independence. Random walk. Law of Large Numbers. Monte Carlo simulation. Central Limit Theorem. Brownian motion. The geometric Brownian motion model of a stock price. Some remarks on portfolio theory.

Exercises: Chapter 3.1: 1, 2, 3, 4, 5, 6, 7, 8, 9

Week 16, 6 hours

Heat conduction, simple random walk, and Brownian motion. Probabilistic representations of solutions to the heat equation and some other parabolic differential equations. The Black-Scholes model and differential equation.

Exercises: Chapter 3.1: 10, 12, 13; Chapter 3.2: 3; Chapter 4.1: 1; Chapter 4.3: 1, 2, 3

Week 17, 6 hours

More on the Black-Scholes model. Call prices. European and American put prices. Simple currency derivatives. Options on forward contracts. Greeks and sensitivity analysis. The Black-Scholes prices of path-dependent options.

Exercises: Chapter 5.1: 1, 3; Chapter 5.2: 1, 2, 3, 4, 5, 6

Week 18, 4 hours

Implied volatility. Several sources of randomness. Bivariate Brownian motion and option pricing.

Exercises: Chapter 5.3: 1, 2, 3, 4, 5; Chapter 5.4: 2, 4; Chapter 5.5: 1

Week 19, 6 hours

More on bivariate Brownian motion and option pricing. A mean-variance approach to portfolio selection.

Exercises: Chapter 3.1: 20; Chapter 6.1: 1, 2, 3; Chapter 3.1: 14, 15, 16, 19

Week 20, 6 hours

Options on dividend-paying stocks.

Exercises: Chapter 6.2: 1, 2, 3; Chapter 6.3: 2; Chapter 7.1: 1, 2, 4

EXAMINATION

Assignments handed in to the examiner

A number of exercises solved and handed in at the latest Friday, March 30 at 15⁰⁰ will result in a maximum of 0.5 point at the final examination in May 2012.

The student may hand in solutions together with at most two other students. However, (due to technical problems) you cannot send your assignments by e-mail. Finally, do not forget to pagenate and write your name(s) on each side (if possible, do not put your solutions in a plastic case).

Written final examination (4 hours)

May 21, 2012, morning, v

September 1, 2012, morning, v

January 2012, ?

Aid not permitted.

The test comprises 15 points (in May 2012 plus the credit from the assignments) and to pass at least 6 points are required (at GU a result greater than or equal to 11 points is graded VG; at Chalmers a result greater than or equal to 9 points and smaller than 12 points is graded 4 and a result greater than or equal to 12 points is graded 5).

At least 6 points are of theoretic nature and at least 3 of these are chosen from the following list:

Theorem 1.1.2; Theorem 1.1.3; Theorem 1.1.4; Theorem 2.1.1; Theorem 2.2.1; Theorem 3.1.1; Theorem 3.3.1; Theorem 4.1.1; Theorem 4.2.1; Theorem 4.3.1; Theorem 4.3.2; Theorem 5.1.1; Theorem 5.2.1; Theorem 5.3.1 (only the formula for delta); Theorem 6.1.1.

Göteborg, March 2, 2011

Christer Borell