

**OPTIONS AND MATHEMATICS** (CTH[mve095], GU[MMA700])  
**ASSIGNMENTS 2013**

(must be handed in at the latest Thursday, April 25, 2013 at 15<sup>00</sup>)

1. (Dominance principle) Suppose  $K, T > 0$  and the prices  $S(0)$ ,  $c(0, S(0), \frac{K}{2}, T)$ ,  $c(0, S(0), K, T)$ , and  $c(0, S(0), \frac{3K}{2}, T)$  are known. Find the time zero price of a European-style derivative paying

$$Y = \max(0, \frac{1}{2}K - |S(T) - K|)$$

at the termination date  $T$ .

2. (Multi-period binomial model,  $d < r < u$ ) Suppose  $\alpha$  is a real number. A financial European-style derivative has the payoff

$$Y = S(T)^\alpha$$

at time of maturity  $T$ . (a) Find  $\Pi_Y(t)$  for  $t = 0, \dots, T - 1$ . (b) The portfolio strategy  $h$  replicates  $Y$ . Find  $h(0) = (h_S(0), h_B(0))$ .

3. Suppose  $A$  and  $B$  are finite unions of subintervals of the real line and let  $X$  and  $Y$  be a random variables. Show that the random variables  $1_A(X)$  and  $1_B(Y)$  are positively correlated if and only if the random variables  $1_{A^c}(X)$  and  $1_{B^c}(Y)$  are positively correlated.

4. Suppose  $D = \{(x, y); 0 < x < 1 \text{ and } y > 0\}$  and let  $(X, Y)$  be a random vector in the plane with the density function

$$f(x, y) = 1_D(x, y)(3x^2y + 1)e^{-y}/2.$$

For which real  $t$  is the variance  $\text{Var}(X - tY)$  minimal?

5. Suppose  $X, Y \in N(0, 1)$  are independent. Find the characteristic function of the random variable  $X(Y + 1)$ .