Hints for recommended exercises in Re.f [2], Sec. 1.1

To solve the exercises, the following tools are required:

- Dominance principle (Ref. [1], pag. 22)
- Ref. [2], Theorem 1.1.1

Exercise 2

As $S(t) = B(t)$, then $S(t) = S(0)e^{rt}$. Take a portfolio which is long one share of the call. The value at maturity is $V(T) = (S(T) - K)_+ = (S(0)e^{rT} - K)_+$. Hence $S(0)e^{rT} < K$ implies $V(T) = 0$ and thus by the dominance principle $V(t) = C(t, S(t), K, T) = 0$. This proves (a) and (b), (c) are proved likewise.

Exercise 5

Consider a portfolio which is long 1 share of the call with strike K_0 and short 1 share of the call with strike K_1

Exercise 6

For the first claim, consider a portfolio that is long 1 share of the call and short 1 share of the stock. By this claim and the put-call parity

$$S(t) - Ke^{-r(T-t)} \leq C(t, S(t), K, T) \leq S(t),$$

which yields the second claim.

Exercise 7

Consider a portfolio which is long 1 share of the call with strike K_1, short 1 share of the call with strike K_0 and long $\frac{K_1 - K_0}{B(T)}$ shares of the risk-free asset.

Exercise 8

Follows by the fist claim in Exercise 6 and the fact that the price of the call is nonnegative

Exercise 9

Differentiate the put-call parity.