Information on Assignment 2, Maintenance planning MVE165 Applied Optimization

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Given below is a mathematical model for finding a schedule that minimizes the costs of maintaining a system during a limited time period. The system consists of several components with economic dependencies.

Implementations of the model in AMPL can be found on the course homepage:
`http://www.math.chalmers.se/Math/Grundutb/CTH/mve165/0708/`
Study the AMPL files carefully to get some hints before you start solving. Call AMPL/CPLEX using the command '`ampl uh.run`'. The file `uh.run` should be edited in order to solve the different instances of the model, as described in the exercises below.

To pass the assignment you should (in groups of two (or one) persons) `(i)` write a report (maximum six pages) that discusses the issues presented in the exercises and questions below. We also ask you to estimate the number of hours you spent on this assignment and write this in the report. The report must be e-mailed to `anstr@chalmers.se` (in pdf format)

at latest on Monday 28 of April 2008.

You should then `(ii)` write an opposition (maximum 1/2 page) to another group's report which must be handed in

at latest on Monday 5 of May 2008.

Questions 1–3 below are mandatory. Students aiming at grade 3 must answer at least one of the questions 4–6, while students aiming at grade 4 or 5 must answer all the questions.

The mathematical model

Sets and parameters

- $\mathcal{N}$ = the set of components in the system. `(in AMPL: Components)`
- T = the number of time steps in the planning period. `(in AMPL: T)`
- T_i = the life of a new component of type $i \in \mathcal{N}$ (measured in number of time steps). It is assumed that $2 \leq T_i \leq T - 1$. `(in AMPL: U)`
- c_{it} = the cost of a replacement component of type $i \in \mathcal{N}$ at time t (measured in €). For some instances it is assumed that c_{it} is constant over time, i.e., $c_{it} = c_i$, $t = 1, \ldots, T$. `(in AMPL: c)`

- d_t = the cost for a maintenance occasion at time t (measured in €). For some instances it is assumed that d_t is constant over time, i.e., $d_t = d$, $t = 1, \dots, T$. `(in AMPL: d)`

Decision variables

- $x_{it} = \begin{cases} 1 & \text{if component } i \text{ is replaced at time } t, \\ 0 & \text{otherwise,} \end{cases} \qquad i \in \mathcal{N}, \quad t \in \{1, \dots, T\}.$

- $z_t = \begin{cases} 1 & \text{if maintenance is made at time } t, \\ 0 & \text{otherwise,} \end{cases} \qquad t \in \{1, \dots, T\}.$

The model

$$
\begin{array}{llcll}
\text{minimize} & \displaystyle\sum_{t=1}^{T-1} \left(\sum_{i \in \mathcal{N}} c_{it} x_{it} + d_t z_t \right), & & & (1) \\
\text{subject to} & \displaystyle\sum_{t=\ell}^{T_i+\ell-1} x_{it} & \geq & 1, \quad \ell = 1, \dots, T - T_i, \quad i \in \mathcal{N}, & (2) \\
& x_{it} & \leq & z_t, \quad t = 1, \dots, T-1, \quad i \in \mathcal{N}, & (3) \\
& x_{it}, z_t & \in & \{0,1\}, \quad t = 1, \dots, T-1, \quad i \in \mathcal{N}. & (4)
\end{array}
$$

Description of the model

(1) The objective is to minimize the total cost for the maintenance during the planning period (the time steps $1, \dots, T-1$) `(in AMPL: Cost)`.

(2) Each component i must be replaced at least once within each T_i time steps `(in AMPL: ReplaceWithinLife)`.

(3) Components can only be replaced at maintenance occasions `(in AMPL: ReplaceOnlyAtMaintenance)`.

(4) All the variables are required to be binary.

Exercises to perform and questions to answer

1. (a) Solve the model (1)–(4) as implemented in `uh-small.mod` with data from `uh-small.dat` and with integer requirements on the variables x_{it} and z_t.
   (b) Relax the integrality requirements and resolve the model. Compare the solutions obtained and discuss their interpretations.
   (c) Add the constraint in `cgcut.mod` to the model `uh-small.mod` and solve (with data from `uh-small.dat`) with *no integrality requirements on the variables*. Compare the solution to those obtained in 1a and 1b above. Explain what happened.

2. Solve four instances of the model (1)–(4) as implemented in `uh-larger.mod` and `uh-larger.dat`. Let the cost of a maintenance occasion for all t be given by $d_t = 0.00001$, $d_t = 20$, $d_t = 100$, and $d_t = 10000$, respectively.

(a) Compare the number of maintenance occasions, the number of replaced components, and the total cost for each of these values of d_t.

(b) Draw illustrating maintenance schedules for each of the solutions.

(c) Discuss what is governing the compromises. What happens between $d_t = 100$ and $d_t = 10000$? Why?

3. Solve the model (1)–(4) as implemented in `uh-larger.mod` and `uh-larger.dat` (with $d = 20$).

   (a) Vary the time horizon between $T = 50$ and $T = 300$ in steps of 50 (refine the step size if/where needed) and draw a graph of the solution time (in CPU seconds) as a function of T.

   (b) Make an analogous graph for the case when the integrality requirements on the variables are relaxed. Denote whether the solutions found are integral or not.

   (c) Comment on the (complexity) properties of the functions in 3a and 3b.

4. (a) In the model (1)–(4) there is no option to perform maintenance at time $t = T$. Explain why there always exists an optimal solution with $x_{iT} = 0$, $i \in \mathcal{N}$, and $z_T = 0$.

   (b) Assume that $T_i = 1$ for some component $i \in \mathcal{N}$. What can be said about the solution to the model (1)–(4)?

   (c) Assume that $T_k \geq T$ for some component $k \in \mathcal{N}$. What can be said about the solution to the model (1)–(4)?

   (d) Cplex uses the branch–and–bound algorithm. On what does it seem to spend most of the solution time: finding an optimal solution or verifying its optimality? Check, e.g., for the instance in exercise 3 with $d_t = 20$ and $T = 200$.

5. Assume that it is required that the system (including all of its components) has a remaining life which is at least $r > 0$ time steps at the end of the planning period (i.e., at time $t = T$).

   (a) Add and/or modify constraints to/in the model to accomplish this and solve the resulting model. Use the model in `uh-larger.mod` with data from `uh-larger.dat` (with $d = 20$ and $T = 155$). Verify that the solution fulfills the stated requirement.

   (b) For relevant values of r (at least two, at most five different values), compare the total cost for maintenance according to this schedule with that of the "original" one. Comment on the number of maintenance occasions and the number of replaced components and compare to the corresponding numbers from the "original" model.

   (c) Which values of r are relevant for this study and why?

6. Assume that there is an option to develop *one* of the components of the system such that its life T_i is prolonged with p_i time steps at a cost of q_i (€). The cost of a (developed) replacement component will then be $c_i + g_i$ (€). Assume that there are six equivalent systems which should be maintained during four time periods, each of length T. Which of the components would be beneficial to develop? Draw your conclusions based on the solution of your modified version of `uh-larger.mod` with data from `uh-larger.dat` and `prolong.dat` (use $d = 20$ and $T = 155$).