Chalmers University of Technology — MVE165
University of Gothenburg — MMG630
Mathematical Sciences — Applied Optimization
Optimization — Assigmnent information
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Assignment 1: Resource theory

Given below is a mathematical model for minimizing the costs of meeting climate targets. An implementation of the model in AMPL can be found on the course homepage:
`http://www.math.chalmers.se/Math/Grundutb/CTH/mve165/0809/`

To pass the assignment you should (in groups of two persons) give satisfactory answers to the following questions in a written report. The report should be 5–7 pages long including illustrating diagrams and it should be

handed in electronically at latest on Friday 3 of April 2009

to anstr@chalmers.se.

In each of the eight questions below the variation of data and/or constraints shall be made starting from the original data (resurs-data.dat) and model (resurs-modell.mod), respectively. Use the case of 450 ppm (see the file resurs-data.dat) as basic scenario for the maximum allowed value of the yearly amount of emissions.

Questions

1. What is the global discounted cost for the energy system if there is no climate policy in place? Compare to the cost of stabalizing the atmospheric carbon content of 400, 450 and 500 ppm. What is the abatement cost for these scenarios (the difference between the total cost of a stabilization scenario and of the baseline[1] scenario)?

2. Compare the total use of oil and coal during the 21th century for the baseline scenario and the three stabilization scenarios. Explain the differences. Reduce the oil reserves by 50% in the baseline scenario. Compare the accumulated carbon dioxide emissions to the original baseline scenario. Explain the difference. What implication does the depletion of the oil reserves have on the severity of the climate change problem?

3. Compare the energy supply for residential heat, industrial heat, electricity, and transport for the baseline and for a 450 ppm scenario. Draw appropriate illustrating graphs in Matlab. In which sectors does carbon dioxide abatement take place first? Explain why.

[1] "Baseline" means "no bounds on the amount of emissions".

4. Increase the fraction of solar that can be used for residential heat to 0.8 and reduce the possibility of using biomass for industrial heat to 0.2. How does the biomass usage in the transport sector change? Compare for the whole century. Explain why the use of biomass in the transport sector changes.

5. Vary the yearly potential for energy supply from biomass fuel (increase with 50%–100%). What is the effect on the energy converted to electricity and transport fuel (E_{xyt})? What is the cost per EJ/year for *reducing* the potential for energy supply from biomass fuel (inflate the numbers for each time step with the discount rate r)? Draw illustrating diagrams (use Matlab).

6. What are the *reduced costs* for extracting fuels and for the investment in equipment for converting energy from solar and biomass to electricity? What does this mean?

7. Increase the potential use of nuclear power from 20% of the electricity demand to 90%. What would be the impact on the transport system? How does the *shadow price* for carbon dioxide emissions change? Compare for the whole century.

8. Increase the investment costs for converting energy from solar$_{\mathrm{H2}}$ to all types of output energy by 100%. What is the effect on the energy supply? What is the effect on the shadow prices for emissions? Compare for the whole century (inflate the numbers for each time step with the discount rate r). Discuss and (try to) explain.

The mathematical model

Sets

- The set of energy inputs (in AMPL: ENERGY_IN):
  $\mathcal{X} = \{\text{solar}_{\text{H2}}, \text{oil}, \text{coal}, \text{solar}, \text{bio}, \text{nuc}, \text{elec}, \text{res}, \text{ind}, \text{transp}\}$.
- The set of energy outputs (in AMPL: ENERGY_OUT):
  $\mathcal{Y} = \{\text{elec}, \text{res}, \text{ind}, \text{transp}\}$, where "elec"=electricity, "res"=residential heat, "ind"=industrial heat, "transp"=transport.
- The set of fuels (in AMPL: FUELS): $\mathcal{F} = \{\text{oil}, \text{coal}, \text{bio}, \text{nuc}\}$.
- The set of time steps: $\mathcal{T} = \{1, \ldots, 13\}$ corresponding to the years[2] $\{2000, 2010, \ldots, 2100, 2110, 2120\}$ (the index 0 represents year 1990) (in AMPL: YEARS).

Parameters

- D_{yt} = yearly demand of energy output y in time step t, $y \in \mathcal{Y}$, $t \in \mathcal{T}$ [EJ/year] (in AMPL: dem[eo,t]).
- δ_{xy} = maximum allowed value for energy conversion per year [EJ/year].
- Q_{xy} = usage of energy the year 2000 (amount of energy converted from type x to type y year 2000) [EJ/year] (in AMPL: init_energy[ei,eo]).
- P = yearly potential for energy supply from biomass fuel [EJ/year] (in AMPL: bio_pot).
- R_f = stock of fossil fuel of type f (EJ) (in AMPL: supply_pot).
- η_{xy} = efficiency of converting energy from type x to type y, $x \in \mathcal{X}$, $y \in \mathcal{Y}$ (in AMPL: effic[ei,eo]).
- ℓ_{xy} = load factor (usage level) of the capital stock per year for converting energy from type x to type y (in AMPL: lf[ei,eo]).
- γ_{xyt} = maximum allowed capital investment in equipment for energy conversion from type x to type y in time step t [TW/year].
- α_{xy} = factor constraining energy conversion with respect to demand (in AMPL: limits).
- τ = life of capital [years] (in AMPL: life_plant).
- r = discount rate (in AMPL: r).
- M = millions of seconds in one year (= 31.536) (in AMPL: Msec_per_year).

[2] The time period studied is 2000–2100. In order to avoid errors due to boundary conditions, the time horizon for the computations is, however, extended to the year 2120.

- $\lambda =$ length of time steps in years ($= 10$) `(in AMPL: t_step)`.
- $p_f =$ cost for extracting fuel of type f [GUSD/EJ] `(in AMPL: price[f])`.
- $k_{xy} =$ investment cost for equipment for converting energy from type x to type y [GUSD/TW] `(in AMPL: cost_inv[ei,eo])`.
- $v_x =$ capital cost for vehicles, energy type x [GUSD/EJ] `(in AMPL: veh_cost[ei])`.
- $\beta_f =$ carbon emission from fossil fuel type f [Mton C/EJ] `(in AMPL: emis_fact[f])`.
- $\overline{\Omega}_t =$ maximum allowed value of the yearly amount of emissions in time step t [Gton C/year] `(in AMPL: emis_up[t])`.

Variables

- $S_{ft} =$ yearly primary supply of fuel f in time step t [EJ/year] `(in AMPL: supply_1[f,t])`.
- $E_{xyt} =$ the yearly amount of energy converted from type x to type y in of time step t [EJ/year] `(in AMPL: en_conv[ei,eo,t])`.
- $e_{yt} =$ the yearly amount of electricity used as secondary input for output energy of type y in time step t [EJ/year] `(in AMPL: extra_dem[eo,t])`.
- $g_{yt} =$ the yearly supply of input energy for output energy of type y in of time step t [EJ/year] `(in AMPL: en_supply[eo,t])`.
- $C_{xyt} =$ yearly capital stock in time step t for energy conversion from type x to type y [TW] `(in AMPL: capital[ei,eo,t])`.
- $I_{xyt} =$ the yearly capital investment in time step t in equipment for energy conversion from type x to type y [TW/year] `(in AMPL: cap_invest[ei,eo,t])`.
- $U_t =$ yearly carbon emission in time step t [Gton C/year] `(in AMPL: C_emission[t])`.
- $\Pi_t =$ the annual cost in time step t [GUSD/year] `(in AMPL: annual_cost[t])`.

The mathematical model

$$\text{minimize} \quad \lambda \sum_{t\in\mathcal{T}} \left[(1+r)^{-(t-1)\lambda}\right] \Pi_t, \tag{1}$$

subject to

$$\begin{aligned}
\sum_{f\in\mathcal{F}} p_f S_{ft} + \sum_{x\in\mathcal{X}} \left(\sum_{y\in\mathcal{Y}} k_{xy} I_{xyt} + v_x E_{x,\text{transp},t} \right) &= \Pi_t, && t\in\mathcal{T}, && (2)\\
S_{\text{bio},t} &\le P, && t\in\mathcal{T}, && (3)\\
\lambda \sum_{t\in\mathcal{T}} S_{ft} &\le R_f, && f\in\{\text{oil},\text{coal},\text{nuc}\}, && (4)\\
\sum_{y\in\mathcal{Y}} E_{fyt} &= S_{ft}, && f\in\mathcal{F}, t\in\mathcal{T}, && (5)\\
\sum_{y\in\mathcal{Y}} E_{\text{elec},y,t} &= e_{\text{elec},t}, && t\in\mathcal{T}, && (6)\\
\sum_{x\in\mathcal{X}} \eta_{xy} E_{xyt} - e_{yt} &= D_{yt}, && y\in\mathcal{Y}, t\in\mathcal{T}, && (7)\\
\sum_{x\in\mathcal{X}} E_{xyt} &= g_{yt}, && y\in\mathcal{Y}, t\in\mathcal{T}, && (8)\\
E_{xyt} - M\ell_{xy} C_{xyt} &\le 0, && x\in\mathcal{X}, y\in\mathcal{Y}, t\in\mathcal{T}, && (9)\\
\lambda I_{xyt} + \left[\left(1-\tfrac{1}{\tau}\right)^{\lambda}\right] C_{x,y,t-1} &= C_{xyt}, && x\in\mathcal{X}, y\in\mathcal{Y}, t\in\mathcal{T}, && (10)\\
C_{xy0} &= 0, && x\in\mathcal{X}, y\in\mathcal{Y}, && (11)\\
I_{xyt} &\le \gamma_{xyt}, && x\in\mathcal{X}, y\in\mathcal{Y}, t\in\mathcal{T}, && (12)\\
E_{xy1} &\le Q_{xy}, && x\in\mathcal{X}, y\in\mathcal{Y} && (13)\\
\eta_{xy} E_{xyt} - \alpha_{xy} e_{yt} &\le \alpha_{xy} D_{yt}, && x\in\mathcal{X}, y\in\mathcal{Y}, t\in\mathcal{T}, && (14)\\
E_{xyt} &\le \delta_{xy}, && x\in\mathcal{X}, y\in\mathcal{Y}, t\in\mathcal{T}, && (15)\\
\sum_{f\in\mathcal{F}} \beta_f S_{ft} &= U_t, && t\in\mathcal{T}, && (16)\\
U_t &\le \overline{\Omega}_t, && t\in\mathcal{T}, && (17)\\
S_{ft}, E_{xyt}, e_{yt}, C_{xyt}, I_{xyt}, U_t, \Pi_t &\ge 0, && x\in\mathcal{X}, y\in\mathcal{Y}, f\in\mathcal{F}, t\in\mathcal{T}. && (18)
\end{aligned}$$

Explanation of the mathematical model

(1) Objective: minimize the sum over the years of discounted annual costs `(in AMPL: tot_cost_M)`.

(2) The annual cost is composed by costs for extracting fuels, inverstment costs for energy conversion equipment, and (extra) vehicle costs for energy conversion `(in AMPL: annual_cost_Q)`.

(3) The yearly primary supply of biomass fuel in time step t may not exceed the yearly supply potential `(in AMPL: supply_pot_Q)`.

(4) For fossil fuels (oil, coal, and nuclear) the sum over the whole time period of the yearly primary supply of fuel may not exceed the stock of fossil fuel `(in AMPL: reserves_Q)`.

(5) In each time step, t, the primary supply of fuel f is converted to (four) different energy output types, y (in AMPL: `supply_1_Q`).

(6) In each time step, t, the amount of electrical energy converted to output energy equals the amount of electricity used as secondary input (in AMPL: `supply_2_Q`).

(7) In each time step, t, and for each output type, y, the amount of energy per year converted to type y minus the amount of y used as secondary input equals the demand for y (in AMPL: `energy_demand_Q`).

(8) In each time step the yearly supply of input energy for otput energy type y equals the sum over the input energy types of energy converted (in AMPL: `en_supply_Q`).

(9) The yearly amount of energy converted from type x to type y in time step t many not exceed the effective (w.r.t. load factor) yearly capital stock (in AMPL: `capital_lim_Q`).

(10) For each input type, x, and each output type, y, of energy conversion, the yearly capital stock in time step t equals the discounted (w.r.t. life of capital) yearly capital stock in time step $t-1$ plus the yearly capital investment in time step t (in AMPL: `capital_Q`).

(11) For each input type, x, and each output type, y, of energy conversion, the capital stock in time step 0 (i.e., year 1990) equals 0 (in AMPL: `capital_init_Q`).

(12) The yearly amount of energy converted from input type x to output type y in time step 1 (i.e., year 2000) may not exceed the supply of energy in year 2000 (in AMPL: `q_init_energy`).

(13) The yearly capital stock is limited for certain time steps and types of input energy (in AMPL: `cap_invest_UB_1,...,cap_invest_UB_4`).

(14) The yearly amount of energy converted from input type x to output type y in time step t may not exceed a certain fraction of the yearly demand of output energy y. (in AMPL: `q_limits`).

(15) The yearly amount of energy converted from input type x to output type y in time step t may not exceed the maximum value for energy conversion per year—due to levels of technical development.

(16) The yearly amount of carbon emmision during time step t equals the sum of carbon emission per year from the primary supply of fuels (in AMPL: `emission_Q`).

(17) The amount of carbon emmision in time step t may not exceed the maximum allowed value (in AMPL: `emis_UB`).