Maintenance optimization

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http://www.chalmers.se/math/EN/research/research-groups/optimization

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A talk with Bo Hägg, CEO Swedish Centre for Maintenance Management, I

- Maintenance = selling reliability at the least cost
- Maint-costs/year: 14 000 Billion SEK (EU), 275 Billion (S)
- Maintenance often seen as wasting money
- Maintenance is often performed too often—inspections and condition monitoring often damage the system
- The truth? A well performed maintenance is an investment in availability and security

A talk with Bo Hägg, CEO Swedish Centre for Maintenance Management, II

- Availability in Swedish manufacturing is low
- OEE (overall equipment effectiveness): combination of availability, performance and quality
- Ideally 100% — in practice $\lesssim$ 85%
- Anders Kinnander, Chalmers (2006): in manufacturing OEE is 64 %; realistically it could be 75–80%

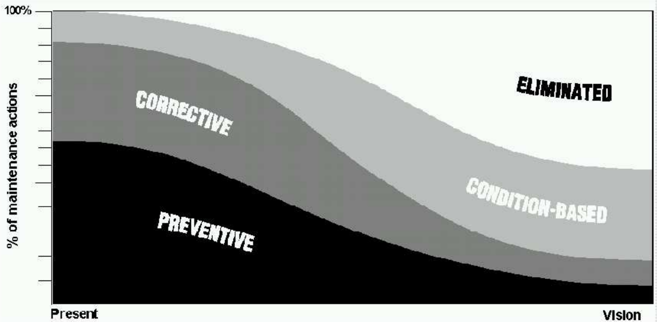
Principles for maintenance

- Preventive maintenance: actions that correct errors before they occur
- Corrective maintenance: actions that correct errors after failure, repairs
- Condition based maintenance: measurements $\longrightarrow$ predictions $\longrightarrow$ actions according to one of the above principles
- Opportunistic maintenance: when maintenance is planned, perform preventive maintenance if it is beneficial

Vision, according to Bo Hägg

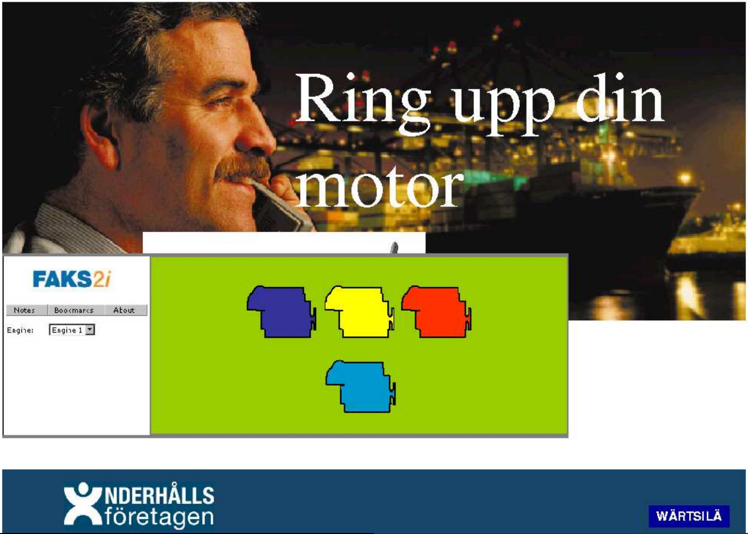
Det Norske Veritas

Tillståndskontroll i Underhållsarbetet



UNDERHÅLLS företagen

Condition based maintenance through your mobile!!



A simple example, I

- System with n parts
- Life of part i : T_i time units
- Time horizon: T time units (eg. contract period)
- Cost of part i at time t: c_{it} monetary units
- Cost for performing any maintenance actions at time t: d_t monetary units

A simple example, II

- Variables in the problem are logical – do something or not
- Modelling using binary variables:

$$x_t = \begin{cases} 1, & \text{if some action is taken at time } t \\ 0, & \text{otherwise} \end{cases}$$

- A decision on an action often implies other necessary actions
- Example: if part i shall be replaced at time t maintenance must be performed and paid for
- Such logical relations are equivalent to linear constraints:

$$\text{if A then B} \quad \Longleftrightarrow \quad x_A \leq x_B$$

A simple replacement problem

- Minimize the total cost of maintaining a working system during a contract period:

Mathematical model

$$
\begin{aligned}
\underset{(x,z)}{\text{minimize}} \quad & \sum_{t=1}^{T} \left(\sum_{i=1}^{N} c_{it} x_{it} + d_t z_t \right), \\
\text{subject to} \quad & \sum_{t=l+1}^{l+T_i} x_{it} \geq 1, && l = 0, \ldots, T - T_i, \quad i = 1, \ldots, N, \\
& x_{it} \leq z_t, && t = 1, \ldots, T, \quad i = 1, \ldots, N, \\
& x_{it} \geq 0, && t = 1, \ldots, T, \quad i = 1, \ldots, N, \\
& z_t \leq 1, && t = 1, \ldots, T, \\
& x_{it}, z_t \in \{0, 1\}, && t = 1, \ldots, T, \quad i = 1, \ldots, N
\end{aligned}
$$

Example, I

- Planning period $T = 7$
- Number of components $|\mathcal{N}| = 3$
- Life of components $T_1 = 3,\ T_2 = 5,\ T_3 = 6$

Replace each component before its life is over

- The components are new at time $t = 0$
- Life of component 1: $T_1 = 3$

$$
\begin{aligned}
x_{11} + x_{12} + x_{13} \quad\quad\quad\quad\quad\quad & \geq 1 \\
x_{12} + x_{13} + x_{14} \quad\quad\quad\quad & \geq 1 \\
x_{13} + x_{14} + x_{15} \quad\quad & \geq 1 \\
x_{14} + x_{15} + x_{16} \quad & \geq 1 \\
x_{15} + x_{16} + x_{17} & \geq 1
\end{aligned}
$$

Example, II

- Life of component 2: $T_2 = 5$

$$\begin{aligned}
x_{21} + x_{22} + x_{23} + x_{24} + x_{25} \qquad\qquad &\geq 1 \\
x_{22} + x_{23} + x_{24} + x_{25} + x_{26} \qquad &\geq 1 \\
x_{23} + x_{24} + x_{25} + x_{26} + x_{27} &\geq 1
\end{aligned}$$

- Life of component 3: $T_3 = 6$

$$\begin{aligned}
x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} \qquad &\geq 1 \\
x_{32} + x_{33} + x_{34} + x_{35} + x_{36} + x_{37} &\geq 1
\end{aligned}$$

- Replace a component at time $t \Rightarrow$ The module is maintained at time t. For $t = 1, \ldots, T$:

$$\left[\begin{array}{ccccc} x_{1t} & & & \leq & z_t \\ & x_{2t} & & \leq & z_t \\ & x_{3t} & & \leq & z_t \end{array}\right] \Leftrightarrow \left[\begin{array}{cccccc} -x_{1t} & & & +z_t & \geq & 0 \\ & -x_{2t} & & +z_t & \geq & 0 \\ & & -x_{3t} & +z_t & \geq & 0 \end{array}\right]$$

- Feasible set: $\left\{ \mathbf{x} \in B^{3 \cdot 7 + 7} \,\middle|\, \mathbf{A}\mathbf{x} \geq \mathbf{b} \right\}$, where $\rightarrow$

The linear system

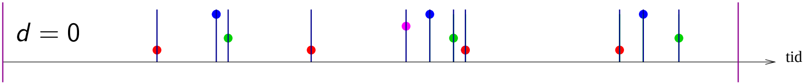
	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		1		x(1,1)
	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		1		x(1,2)
	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		1		x(1,3)
	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		1		x(1,4)
	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		1		x(1,5)
	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		1		x(1,6)
	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		1		x(1,7)
	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0		1		x(2,1)
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0		1		x(2,2)
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0		1		x(2,3)
	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0		0		x(2,4)
	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0		0		x(2,5)
	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0		0		x(2,6)
	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0		0		x(2,7)
	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0		0		x(3,1)
A =	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	b =	0	x =	x(3,2)
	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1		0		x(3,3)
	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0		0		x(3,4)
	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0		0		x(3,5)
	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0		0		x(3,6)
	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0		0		x(3,7)
	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0		0		z(1)
	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0		0		z(2)
	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1		0		z(3)
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	0	0	0	0	0		0		z(4)
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	0	0	0	0		0		z(5)
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	0	0	0		0		z(6)
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	0	0		0		z(7)
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	0		0		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0		0		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	1		0		

Properties

- The integrality restrictions on the replacement ($\mathbf{x}$) variables can be removed. (Argument through a property of A of being totally unimodular.) Hence the number of integer (or, binary) variables equals the number of time periods, T
- Provided that costs are non-increasing, replacements are only performed at some component's life limit
- Given a feasible vector $\mathbf{z}$, an optimal $\mathbf{x}$ is found by solving an LP (from item 1), or by a greedy procedure (from item 2)
- The polytope described by the convex hull of feasible points is full-dimensional
- The problem still is NP-hard

Opportunistic maintenance or not?

- Example case: four parts with different lives and costs
- A replacement of a part is shown as a dot with the resp. colour at the given time



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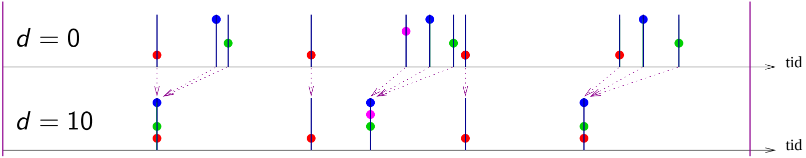
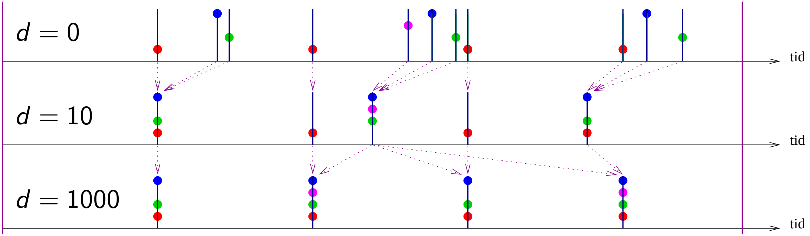


Opportunistic maintenance or not?

- Example case: four parts with different lives and costs
- A replacement is shown as a dot with the resp. colour at the given time
- The bigger the fixed cost the more interesting opportunistic maintenance becomes



Visions

- Wind power: collaboration with KTH, Vattenfall/Lillgrund. Plans for future PhD students together with Energy and Environment (Vindforsk, SKF). Huge interest from companies!
- Nuclear power: collaboration with KTH, Forsmark
- Collaboration plans at Chalmers together with CHARMEC (rail mechanics)
- Other areas: process industry, mechanical industry, logistics, . . .