Optimization models for the maintenance planning of aircraft engines

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Project background

Maintenance of aircraft engines is expensive:

- *spare parts* cost up to 2 Mkr
- *total cost* for maintenance of a jet engine: 15–30 Mkr
- *rent* for a spare engine: 15 kkr/day

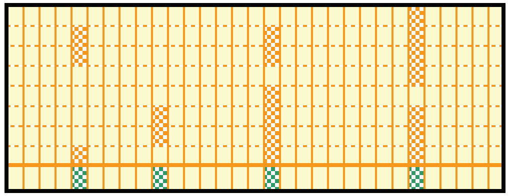
Opportunistic maintenance:

At each maintenance occasion, possible to *perform more maintenance than* what is absolutely *necessary*

⇨ totally fewer maintenance occasions

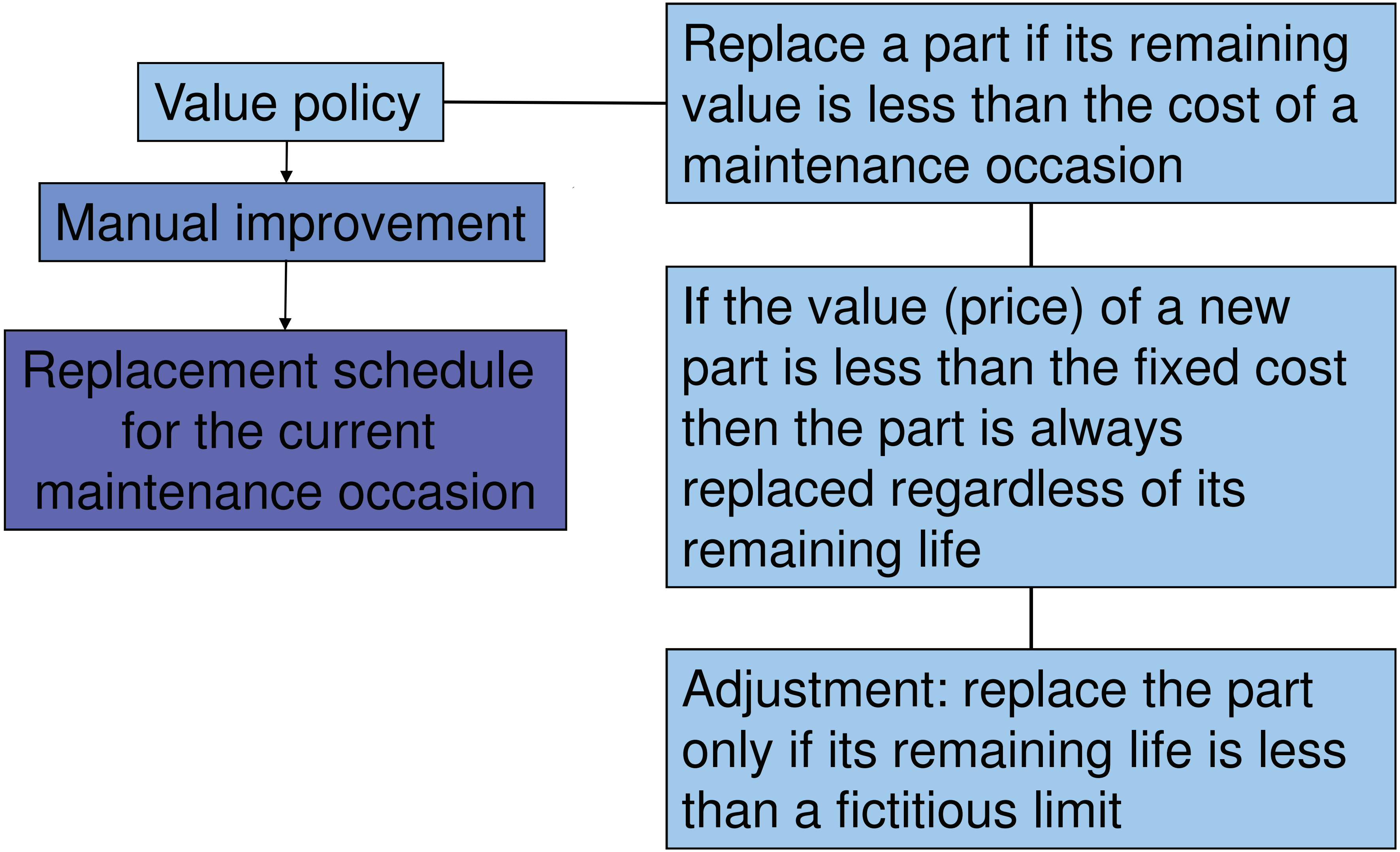
⇨ totally lower cost

The purpose of the project



- Create a *methodology* that generates good *replacement schedules* for components in aircraft engines
- Consider:
  - *Life time restricted* and ”*on condition*”-components
  - *Fixed cost* when an engine/module is taken to the workshop
  - *Work costs* to set free engine modules their components
  - Utilize a *store* of used components
- *Minimize total flight hour cost* during the contract period

VAC:s existing value policy

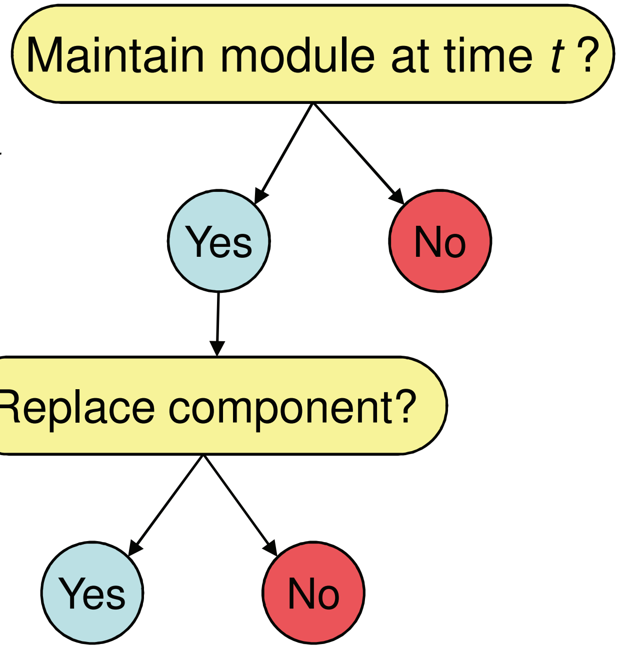


A simple optimization model for the whole contract period

- For each component i in the module:
  - *Cost* of a new component: c_i
  - *Life* of a new component: T_i
  - *Remaining life* of current component: τ_i
- *Contract period* divided into T *time periods* $t = 1,...,T$
- Maintenance possible at start of each time period (*discrete time steps*)
- A *fixed cost* per maintenance occasion: d

A mathematical optimization model for maintenance planning

Definition of variables



$$z_t = \begin{cases} 1 & \text{if the module is maintained at time } t \\ 0 & \text{otherwise} \end{cases}$$

$$x_{it} = \begin{cases} 1 & \text{if component } i \text{ is replaced at time } t \\ 0 & \text{otherwise} \end{cases}$$

Basic mathematical model: one module, N parts, T time steps

$$\text{minimize} \quad \sum_{t=1}^{T} \left(\sum_{i \in N} c_i x_{it} + d z_t \right)$$

$$\text{subject to} \quad \sum_{t=1}^{\tau_i} x_{it} \geq 1, \qquad i \in N, \qquad \text{replace part before its remaining life is over}$$

$$\sum_{t=l}^{T_i+l-1} x_{it} \geq 1, \qquad l = 1, \ldots, T - T_i + 1, \quad i \in N, \qquad \text{replace part at least once in a lifetime}$$

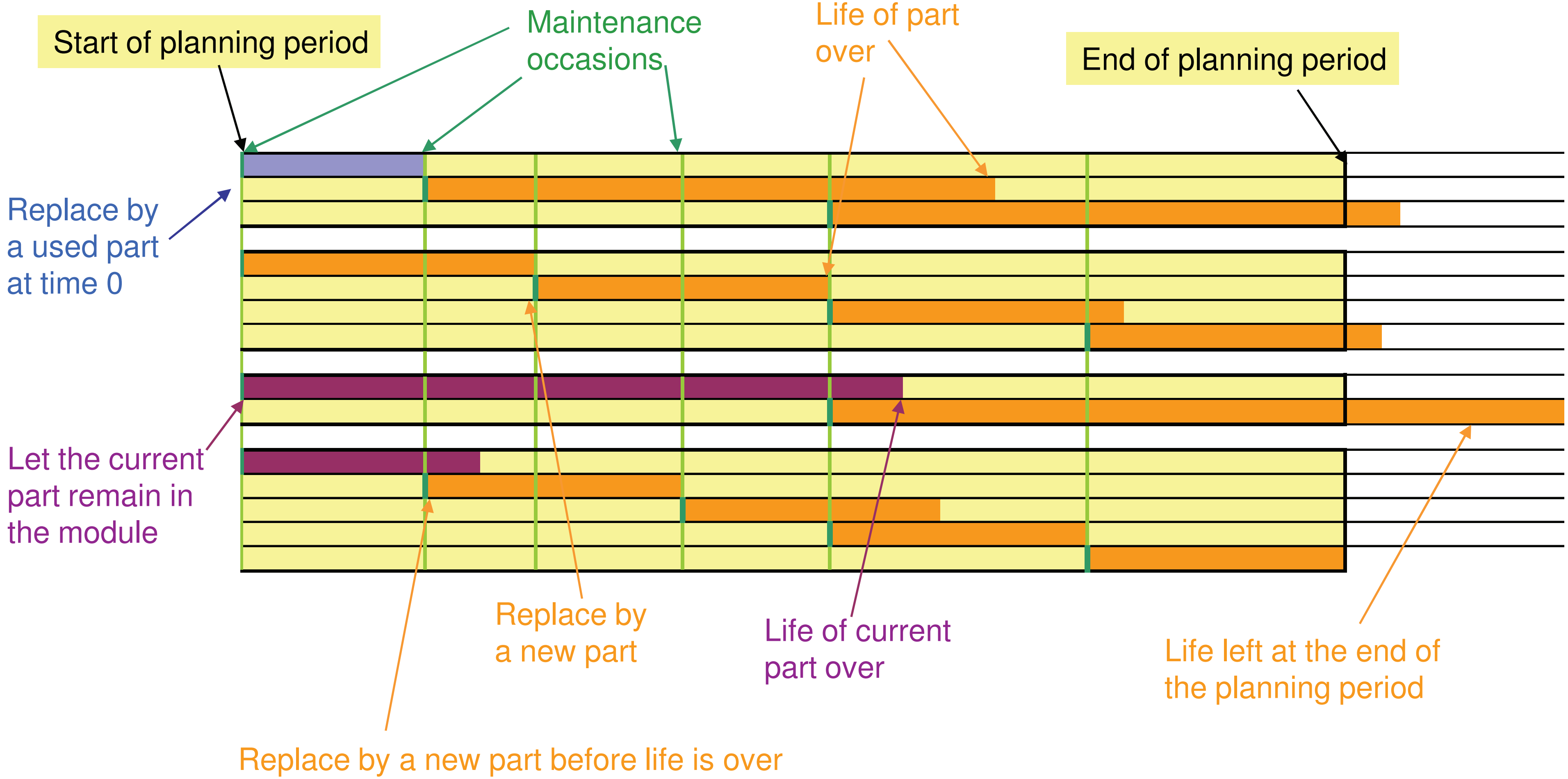
$$x_{it} \leq z_t, \qquad t = 1, \ldots, T, \quad i \in N, \qquad \text{replace part only at maintenance occation}$$

$$x_{it} \in \{0,1\}, \quad t = 1, \ldots, T, \quad i \in N,$$

$$z_t \in \{0,1\}, \quad t = 1, \ldots, T.$$

- $x_{it} \in \{0,1\}$ can be relaxed to $x_{it} \geq 0$ integrality property

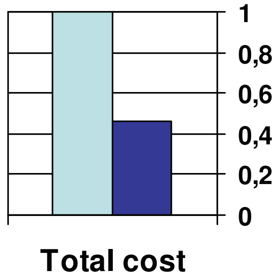
A maintenance schedule for four components in an engine module

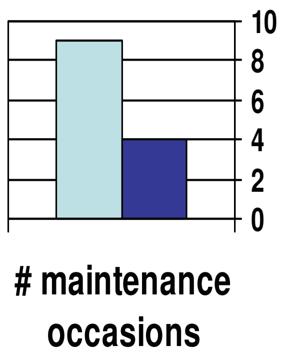


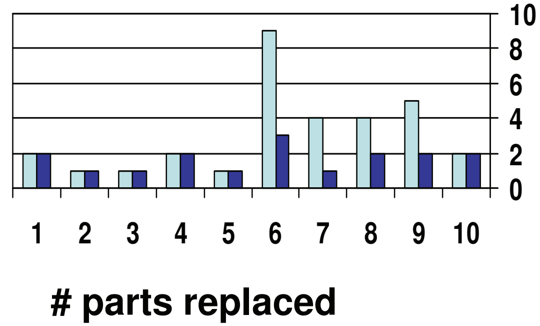
Comparison of the methods

- An engine module with 10 components
- Only life time restricted (deterministic) components

- Value policy
- Optimization







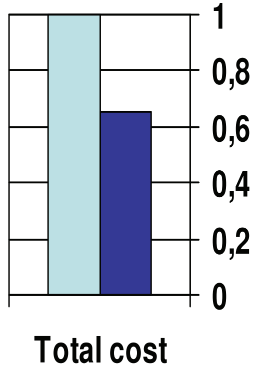
Comparison of the methods using stochastic simulations

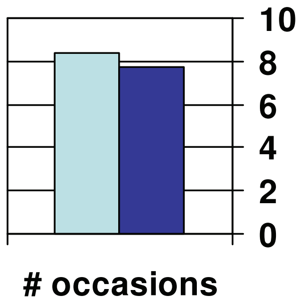
- An engine module with 10 components
- Parts 1, 4, 5, 6, 9, 10 are OC (Weibull)

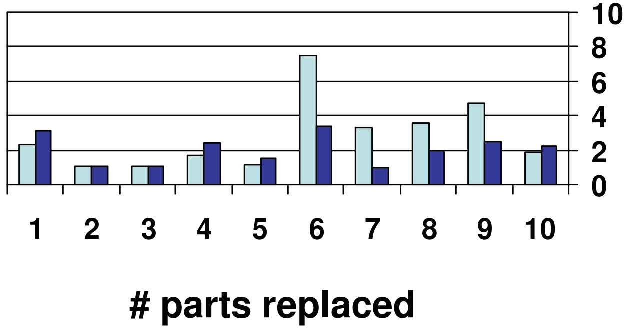
Part no	1	4	5	6	9	10
β	2	2	4	4	6	6

- Value policy
- Optimization

- Average values from 200 scenarios

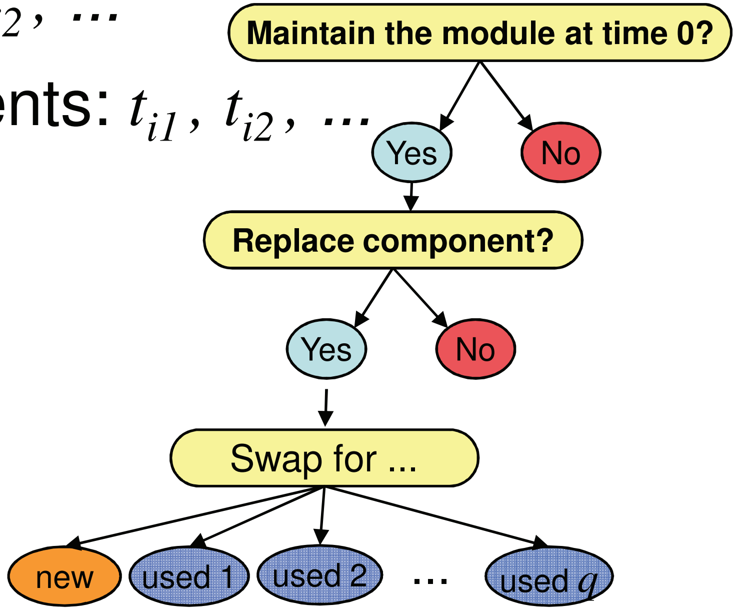






A store of used components

- For each part i in the module there is a *store of used components* at time 0 (at present maintenance occasion):
  - *Costs* for used components: $k_{i1}, k_{i2}, \ldots$
  - *Remaining lives* of used components: $t_{i1}, t_{i2}, \ldots$

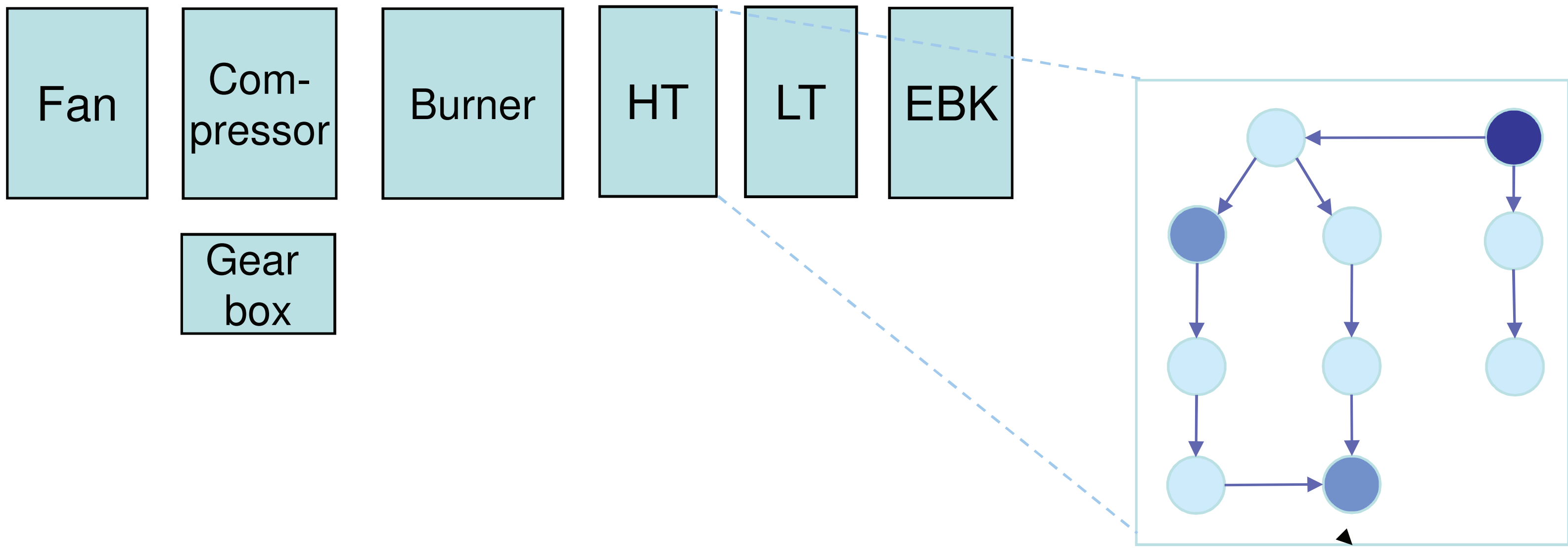


- Additional variables:

$$
s_{ij} = \begin{cases} 1 & \text{if used individual } j \text{ of component } i \\ & \text{from the store is used at time } 0 \\ 0 & \text{otherwise} \end{cases}
$$

Several modules in an engine

- *Work costs to set modules free*
- *Work costs to set components free*



A mathematical model for a whole engine

parameters

$$c_{it}^{m} = \text{price of a spare of part } i \text{ in module } m \text{ at time } t$$

$$\tilde{c}_{ik}^{m} = \text{price for used individual } k \text{ of part } i \text{ in module } m \text{ at } t = 0$$

$$a_{it}^{m} = \text{cost of removing part } i \text{ in module } m \text{ at time } t$$

$$b_{nt} = \text{cost of performing activity } n \text{ at time } t$$

$$d_{t} = \text{fixed cost for maintaining the engine at time } t$$

$$T = \text{length of planning period (\# time steps)}$$

$$T_{i}^{m} = \text{life of new part } i \text{ in module } m$$

$$\tilde{T}_{i}^{m} = \text{remaining life of part } i \text{ in currently in module } m$$

$$e^{mi} = \text{\# used individuals of part } i\text{, module } m \text{ in store at } t = 0$$

$$\bar{T}_{ik}^{m} = \text{remain. life of used indiv. } k\text{, part } i\text{, module } m \text{ in store, } t = 0$$

$$f_{m} = 1 \text{ if maint. of module } m \text{ should be planned,} = 0 \text{ if not}$$

A mathematical model for a whole engine

variables

$x_{it}^m = 1$ if part i in module m is replaced at time t, $= 0$ if not

$u_{ik}^m = 1$ if part i in module m is replaced by used individual k at $t = 0$, $= 0$ if not

$y_{it}^m = 1$ if part i in module m is removed at time t, $= 0$ if not

$z_t^m = 1$ if module m is maintained at time t, $= 0$ if not

$v_{nt} = 1$ if activity n is performed at time t, $= 0$ if not

$w_t = 1$ if the engine is maintained at time t, $= 0$ if not

Tests and results

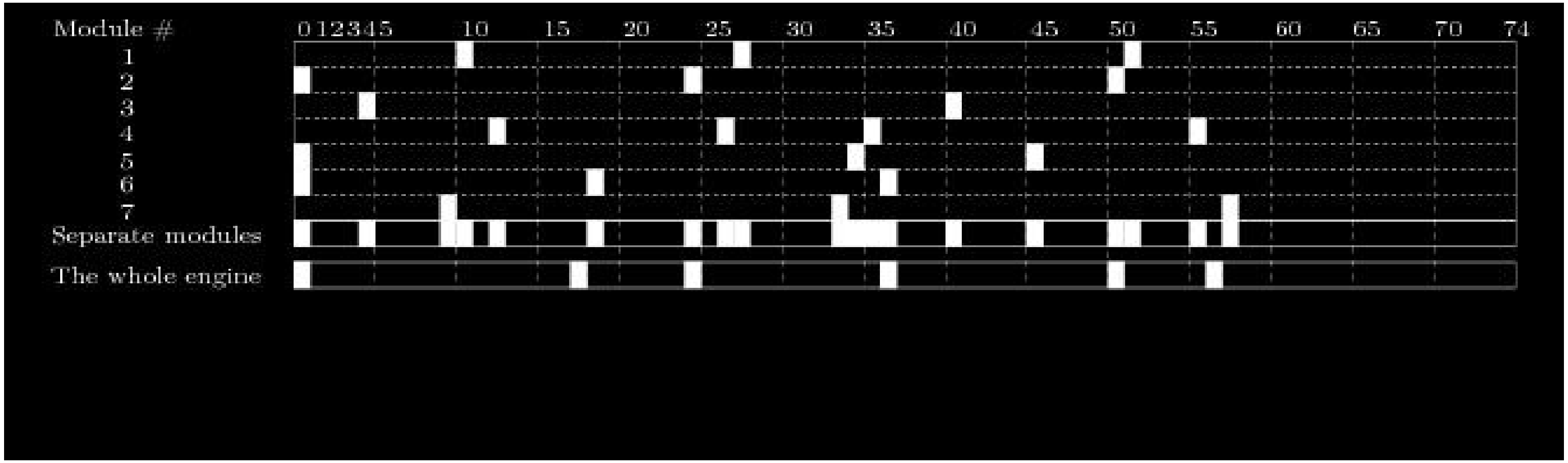
- Discretization: 33,33 flight hours per time step
- Length of the planning period = 2500 flight hours, *T*=75
- Total number of parts in the engine = 61
- Number of modules in the engine = 7
- Number of variables in the model = 10425
- Integrality property for some of the variables
- Number of binary variables in the model = 5775

$$\left(\; 2^{5775} \approx 2.8 \cdot 10^{1738} \; \right)$$

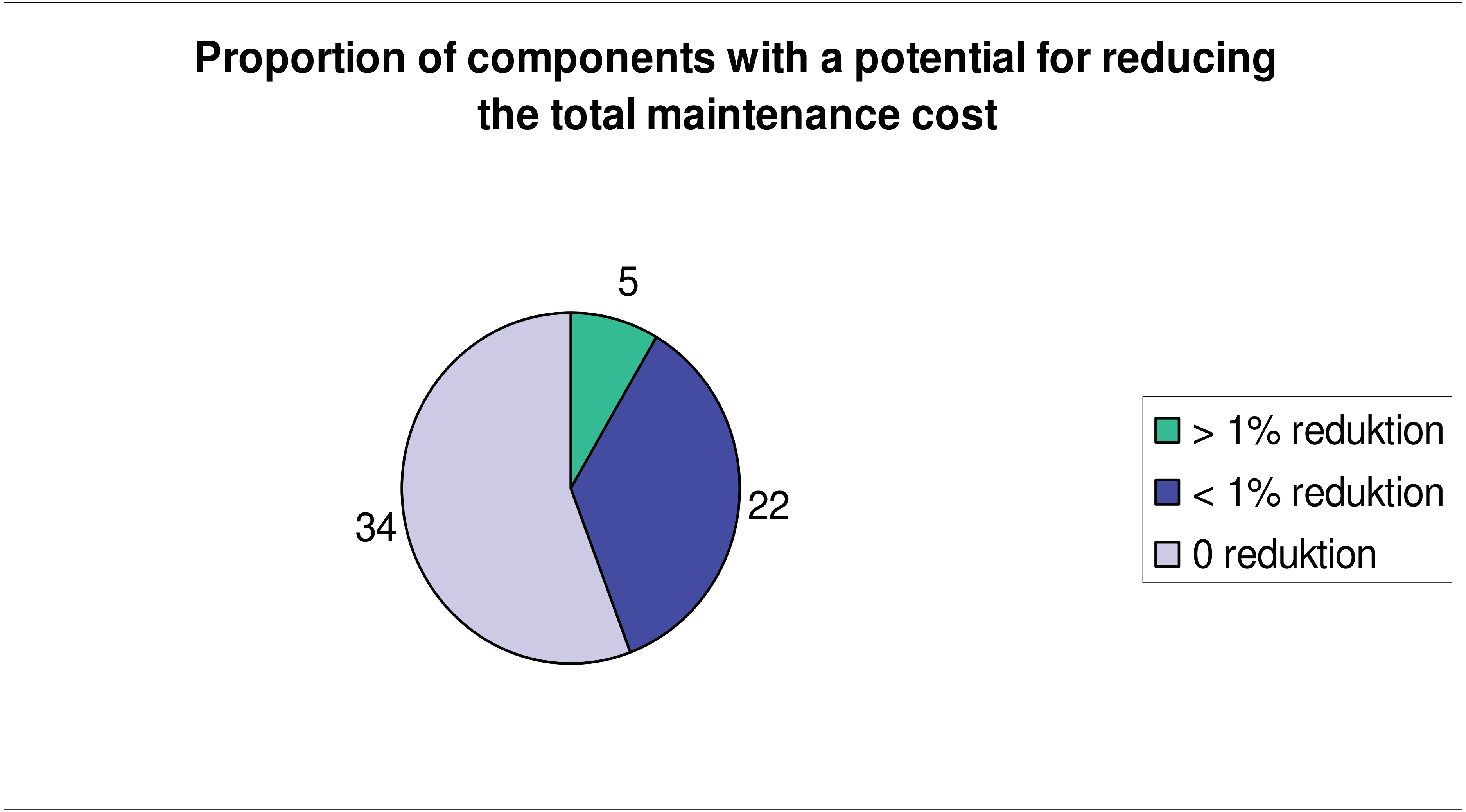
Advantage of simultaneous optimization

An old engine with a store of used spares at t=0

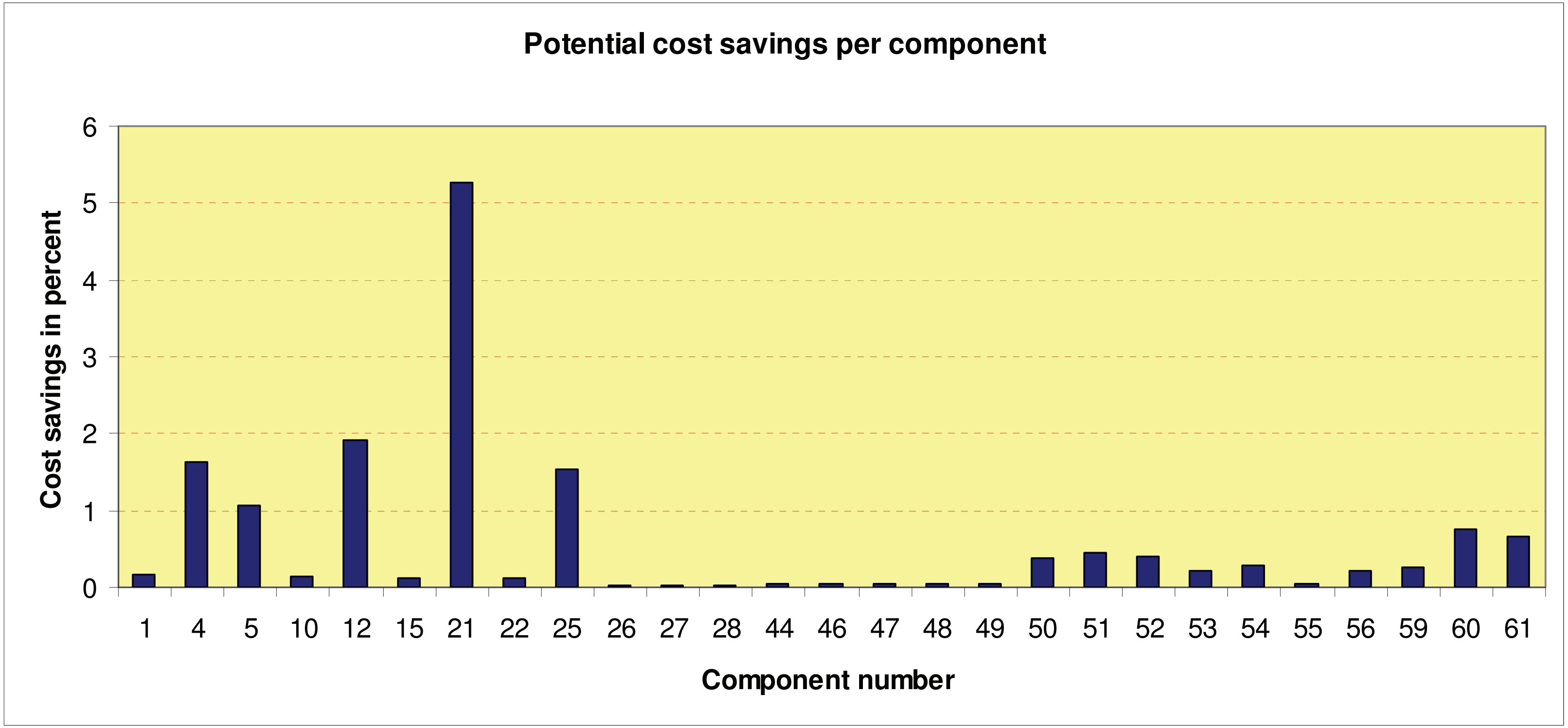
Optimization over:	# maintenance occasions	# replaced parts	Total cost (normalized)	CPU time (sec)
separate modules	19	90	1.222	3.08
the whole engine	6	92	1.000	1.25



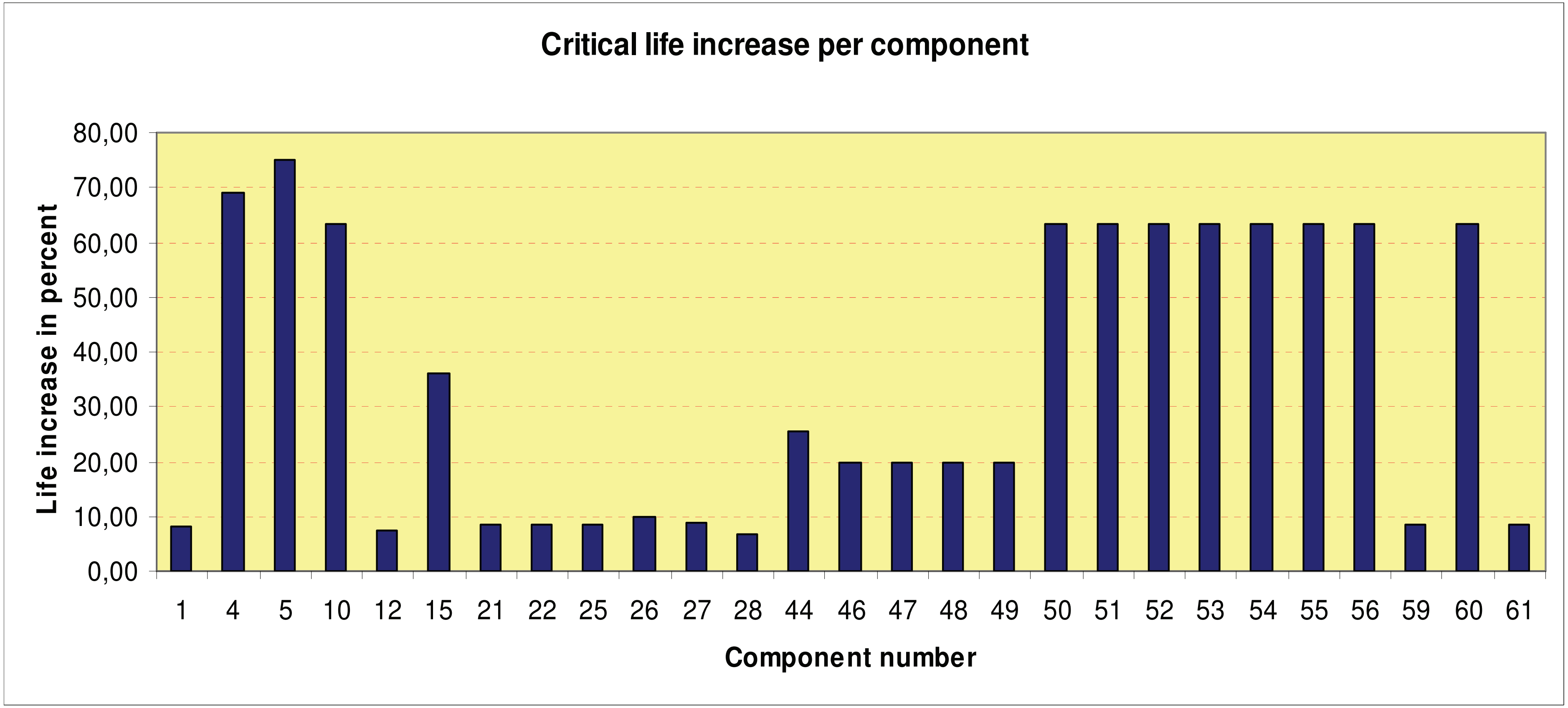
Product development



Product development, continued



Product development, continued



Product development, continued