MVE165/MMG630, Applied Optimization
Lecture 7
Exercises on Linear programming, Network models and Integer linear programming

Anna Nyström

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Exercise 1: The knapsack problem – branch and bound

- Solve the following ILP with branch and bound:

$$\begin{array}{lrcrcrcl} \text{maximize} & 5x_1 & + & 6x_2 & + & 7x_3 & & \\ \text{s.t.} & 2x_1 & + & 3x_2 & + & 3x_3 & \leq & 9 \\ & & & & & x_1, x_2, x_3 & \in & [0, 2] \end{array}$$

- Branch and bound – Main concepts
  - Relaxation: Here LP
  - Branching strategy: Here $\geq$ and $\leq$
  - Tree search strategy: Here Depth-first
  - Node cutting criteria:
    - infeasible solution,
    - optimal solution is feasible in ILP, or
    - optimal value is worse than that of any feasible solution

Exercise 2: Company producing wooden furniture – Sensitivity analysis

A company produces four different sorts of wooden furniture. Three basic operations are made: cutting, shaping and grinding. There are 910 machine hours available for cutting, 800 machine hours for shaping and 480 machine hours for grinding. The company's objective is to maximize the profit. The initial simplex tableau is given by:

	-z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	$\bar{b}$
-z	1	90	160	40	100				0
s_1		7	8	3	5	1			910
s_2		5	4	8	5		1		800
s_3		2	8	4	2			1	480

Exercise 2: Company producing wooden furniture – Sensitivity analysis

After simplex, the optimal tableau is:

	-z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	$\bar{b}$
-z	1	-10		-130		0	-15	-25/2	-18000
s_1		2		-11/2		1	-3/4	-5/8	10
x_4		1		3/2	1		1/4	-1/8	140
x_2			1	1/8			-1/16	5/32	25

Questions:

[a] What are the implicit prices (shadow price/dual value/margin price) for each constraint?

Exercise 2: Company producing wooden furniture – Sensitivity analysis

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	-z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	$\bar{b}$
-z	1	-10		-130			-15	-25/2	-18000
s_1		2		-11/2		1	-3/4	-5/8	10
x_4		1		3/2	1		1/4	-1/8	140
x_2			1	1/8			-1/16	5/32	25

Questions:

[b] If a sawmill offers further machine hours for 10/hour, should you accept? What if another grinding company offers capacity for 10/hour, should you accept? How many hours would you buy?

Exercise 2: Company producing wooden furniture – Sensitivity analysis

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	-z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	$\bar{b}$
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s_1		2		-11/2		1	-3/4	-5/8	10
x_4		1		3/2	1		1/4	-1/8	140
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Questions:

[c] Which profit is needed to make product 3 worth to produce?

Exercise 2: Company producing wooden furniture – Sensitivity analysis

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-z	1	-10		-130			-15	-25/2	-18000
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Questions:

[d] How much can the profit for product 4 change without changing the produced quantities?

Exercise 2: Company producing wooden furniture – Sensitivity analysis

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	-z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	$\bar{b}$
-z	1	-10		-130			-15	-25/2	-18000
s_1		2		-11/2		1	-3/4	-5/8	10
x_4		1		3/2	1		1/4	-1/8	140
x_2			1	1/8			-1/16	5/32	25

Questions:

[e] The managers are considering to introduce a new product. The product demands 5 hours cutting, 3 hour shaping and 2 hour grinding. The expected profit is 120/unit. What is the decision?

Exercise 3: Duality

$$
\begin{array}{lrcrcrcr}
\text{maximize} & z & = & 2x_1 & + & 3x_2 & + & 4x_3 \\
\text{s.t.} & x_1 & + & 2x_2 & + & x_3 & \leq & 4 \\
 & 2x_1 & + & 3x_2 & + & 3x_3 & \geq & 1 \\
 & & & & & & x_1, x_2 \geq & 0
\end{array}
$$

- [a] Give the dual program
- [b] Choose an arbitrary point in the primal and one the dual, respectively, and show that weak duality holds
- [c] Determine the optimal solution for the primal and for the dual

Exercise 4: Network modelling

Two cities, A and B, produce 500 tons and 400 tons, respectively, domestic waste every day. The waste is transported to, and burned up, in one of the two burning plants, C and D, with a capacity of 500 tons per day each. The costs for burning at C is 320 SEK/ton, and at D 240 SEK/ton. During the burn up the waste is converted into dross, and 1 ton waste results in 200 kg dross. The dross is transported to one of the two end plants E or F. Each end plant can take 150 tons of dross/day. There are no costs when dumping the dross. The transportation costs are 150 SEK per ton and mile, for both waste and dross. The distances are given in table below.

	A	B	E	F
C	7	8	1	3
D	1	9	3	2

Formulate a network model to decide minimal cost for transporting and burning the waste, and transporting and dumping the dross.