

Maintenance optimization

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Some current/recent projects at the optimization group

- Modelling/optimization of production and maintenance schedules
- Dynamic optimization of a multi-task cell (Volvo Aero)
- Optimization of investments in process integration (Energy & Environment)
- Stochastic mathematical programs with equilibrium constraints
- Robust biological optimization of radiation therapy (SU)
- Robust construction of traffic toll schedules (GMMC)
- Optimization of truck configurations (Volvo 3P)

Funding

- Chalmers Sustainable Transport Initiative (VINNOVA, Chalmers)
- Chalmers Energy Initiative (The Swedish Energy Agency/Energimyndigheten, Chalmers)
- Optimum scheduling of a multi-task cell (The Swedish Research Council/Vetenskapsrådet)
- Decision support for optimum scheduling of production of aircraft engine components (VINNOVA)
- Development of generic mathematical optimization models and algorithms for the solution of opportunistic and preventive maintenance planning problems in industry (The Swedish Energy Agency/Energimyndigheten)
- Optimization of truck configurations (Volvo 3P)
- GMMC — Gothenburg Mathematical Modelling Centre (Swedish Foundation for Strategic Research/SSF, Chalmers)

Maintenance optimization — a background

- Invitation 2000 from Volvo Aero Corporation (VAC): maintenance of the RM12 jet engine (JAS 39 Gripen)
- Paired PhD project between applied math/optimization and math statistics/material fatigue and reliability
- Optimization student: a model for opportunistic maintenance; superior to simpler policies
- Math statistics student: models for the determination of life distributions based on crack growth
- Continuation projects: VAC; maintenance of components in wind and nuclear power plants

Early literature and inspirations, I

- Research since the 1930s, mostly in isolation
- 1950s: RAND, Santa Monica (Bellman); military applications
- 1960s: Stanford (Wagner); application of scheduling
- Focus: infinite planning horizon, few parts, policies; joint work mathematical statistics & mathematical programming — classic operations research
- Campbell (1941): replacement of lamps along a city street. Two policies can be utilized, where the first is to replace each lamp when it breaks, and the other is to replace all lamps as soon as one breaks
- The first opportunistic replacement model; further development at RAND 1960–
- Exceptional also in that the planning horizon is finite

Inspirations, II

- Our opportunistic model for the finite-horizon case is based on a paper by Dickman, Epstein & Wilamowsky (1981)
- Extended to:
 - stochastic programming models in order to cover non-deterministic life lengths
 - more complex maintenance decisions: replace or repair; utilize a warehouse of used components
 - more complex decisions, including production scheduling
 - more complex component structures: redundancies (k out of n) w/w-o efficiency losses, wear of components from start/stop, work costs for disassembly/assembly
- The paper [DEW81] is remarkably absent from reference lists!

A conversation with Bo Hägg, CEO Underhållsföretagen

- Maintenance = selling reliability at the least cost
- Maintenance costs/year: 14 000 Billion SEK (EU), 275 Billion SEK (S)
- Maintenance is often seen merely as a cost
- Maintenance is typically done too often — inspections and measurements damage the systems
- Truth: well performed maintenance is an investment in availability and safety



Maintenance principles

- Preventive maintenance: actions that prevent failure
- Corrective maintenance: actions after failure, repairs
- Condition based maintenance: measurements → predictions
→ actions according to a maintenance principle
- Opportunistic maintenance: when maintenance must be performed, also perform some preventive maintenance actions

A simple example, I

- System with n parts
- Life of part i : T_i time units
- Horizon: T time units (ex. contract period)
- Cost of part i at time t : c_{it} monetary units
- Cost for performing any maintenance at time t : d_t monetary units

A simple example, II

- Variables are logical – do something or not
- Model uses binary variables:

$$x_t = \begin{cases} 1, & \text{if "something" is done at time } t \\ 0, & \text{otherwise} \end{cases}$$

- A decision often implies other necessary decisions
- Example: if part i shall be replaced at time t maintenance must be performed
- Such logical relations are equivalent to linear constraints:

$$\text{if A then B} \iff x_A \leq x_B$$

The basic replacement problem, I

- Goal: minimize the total cost for a working system during the contract period:

Mathematical model

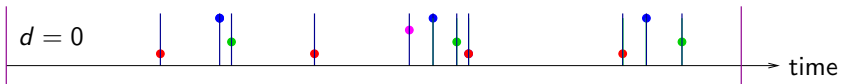
$$\begin{aligned}
 & \underset{(x,z)}{\text{minimize}} && \sum_{t=1}^T \left(\sum_{i=1}^N c_{it} x_{it} + d_t z_t \right), \\
 & \text{subject to} && \sum_{t=l+1}^{l+T_i} x_{it} \geq 1, \quad l = 0, \dots, T - T_i, \quad i = 1, \dots, N, \\
 & && x_{it} \leq z_t, \quad t = 1, \dots, T, \quad i = 1, \dots, N, \\
 & && x_{it} \geq 0, \quad t = 1, \dots, T, \quad i = 1, \dots, N, \\
 & && z_t \leq 1, \quad t = 1, \dots, T, \\
 & && x_{it}, z_t \in \{0, 1\}, \quad t = 1, \dots, T, \quad i = 1, \dots, N
 \end{aligned}$$

The basic replacement problem, II

- The objective is to minimize the total cost of having a working system during the contract period
- The first constraint states that, for any given item i in the system, during any time interval T_i time steps long, the part must be replaced at some point
- The second constraint ensures that we cannot perform the above replacement without paying the fixed cost d_t for performing a maintenance operation; once we do pay, any maintenance action becomes possible at that time
- The remaining constraints ensures that the variables only take meaningful values

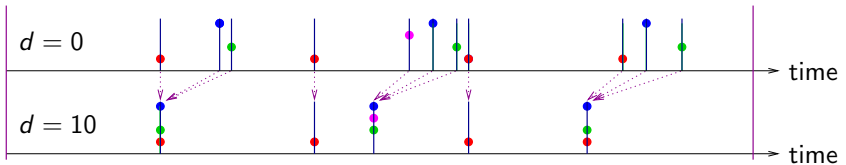
Opportunistic maintenance or not?

- Example: four parts with different prices and lives
- A replacement is marked with a dot; its colour represents the type of part replaced



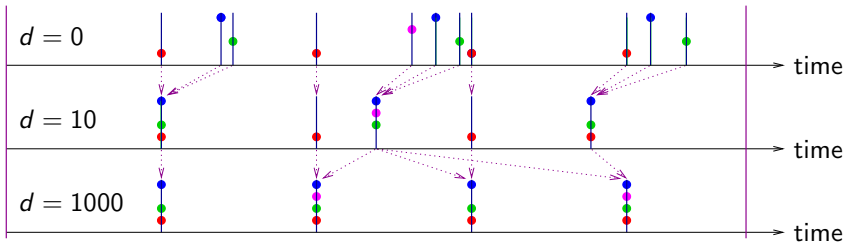
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Opportunistic maintenance or not?

- Example: four parts with different prices and lives
- A replacement is marked with a dot; its colour represents the type of part replaced
- The bigger the fixed cost, the more interesting opportunistic maintenance becomes; also more items are replaced



Example, I

- Planning period $T = 7$
- Number of components $|\mathcal{N}| = 3$
- Life of components $T_1 = 3, T_2 = 5, T_3 = 6$

Replace each component before its life is over

- The components are new at time $t = 0$
- Life of component 1: $T_1 = 3$

$$x_{11} + x_{12} + x_{13} \geq 1$$

$$x_{12} + x_{13} + x_{14} \geq 1$$

$$x_{13} + x_{14} + x_{15} \geq 1$$

$$x_{14} + x_{15} + x_{16} \geq 1$$

$$x_{15} + x_{16} + x_{17} \geq 1$$

Example, II

- Life of component 2: $T_2 = 5$

$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} \geq 1$$

$$x_{22} + x_{23} + x_{24} + x_{25} + x_{26} \geq 1$$

$$x_{23} + x_{24} + x_{25} + x_{26} + x_{27} \geq 1$$

- Life of component 3: $T_3 = 6$

$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} \geq 1$$

$$x_{32} + x_{33} + x_{34} + x_{35} + x_{36} + x_{37} \geq 1$$

- Replace a component at time $t \Rightarrow$ The module is maintained at time t . For $t = 1, \dots, T$:

$$\begin{bmatrix} x_{1t} & \leq & z_t \\ x_{2t} & \leq & z_t \\ x_{3t} & \leq & z_t \end{bmatrix} \Leftrightarrow \begin{bmatrix} -x_{1t} & + z_t & \geq & 0 \\ -x_{2t} & + z_t & \geq & 0 \\ -x_{3t} + z_t & \geq & 0 \end{bmatrix}$$

- Feasible set: $\{\mathbf{x} \in B^{3 \cdot 7 + 7} \mid \mathbf{Ax} \geq \mathbf{b}\}$, where \rightarrow

Property I: the replacement problem is NP-hard

Theorem

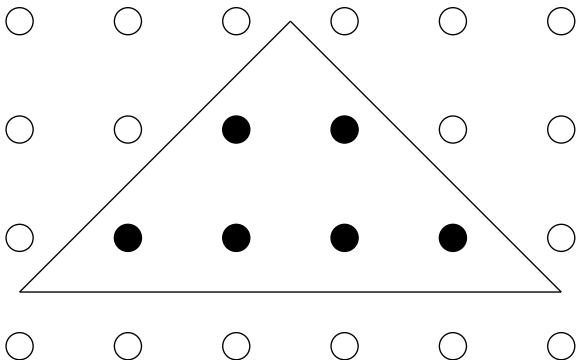
Set covering is polynomially reducible to the replacement problem

Property II: we can relax the integrality requirements on x_{it}

- Totally Unimodular \iff every submatrix $\det \pm 1$
- Constraint matrix TU + integer r.h.s. \implies integer polyhedron
- Consecutive ones + unit matrix \implies TU

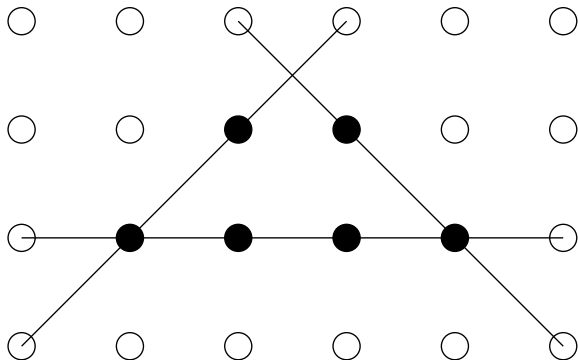
Property III: all inequalities are facet defining

No inequalities are facet defining



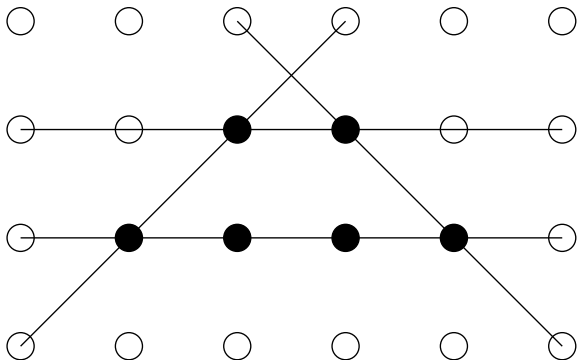
Property III: all inequalities are facet defining

All inequalities are facet defining



Property III: all inequalities are facet defining

Integral polyhedron



Additional facial structure, I

- Small-scale problems from practice can still take 30 hours using CPLEX 12.1
- Cutting planes, reformulations, heuristics?
- Proposal: add further structures implied by the original constraints
- Chvátal–Gomory inequalities, 1: consider $\sum_{i=1}^m a_{ij}x_j \geq b_i$, $i = 1, \dots, m$
- Chvátal–Gomory inequalities, 2: for $\mathbf{u} \geq \mathbf{0}^m$, take

$$\sum_{j=1}^n \left[\sum_{i=1}^m u_i a_{ij} \right] x_j \geq \left[\sum_{i=1}^m u_i b_i \right]$$

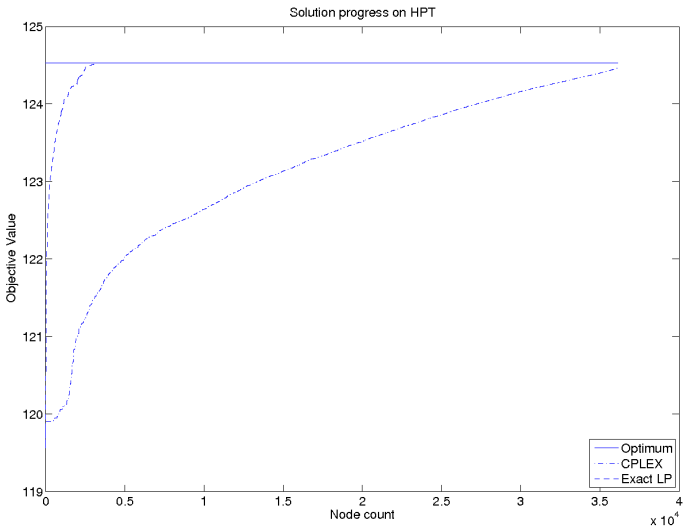
Additional facial structure, II

- If repeated enough, *all* additional facets can be generated
- In fact the above still holds with $u_i \in \{0, \frac{1}{2}\}$
- But: still too complicated
- For our problem: (i) must use an odd number of life constraints; (ii) integrality means we must mix life and maintenance constraints
- Similar to odd cycle inequalities for set covering/packing problems
- Such constraints can be generated as solutions to graph problems, where nodes correspond to an odd number of life constraints, and arcs link nodes when two constraints are mixed, or overlap in time and belong to the same component

Additional facial structure, III

- These are facets!
- Improves LP bound especially when $\min_i T_i$ is small
- But: still too complicated
- A Markov chain type graph structure has been built such that an optimal scheme corresponds to a shortest path in the graph
- Too huge to solve with Dijkstra; use approximate dynamic programming with memory restrictions; essentially means that we use LP-relaxation to produce bounds on the remaining costs
- Good idea for the future solution of stochastic versions; only the today decision need be accurate
- Beats CPLEX on most instances tested
- Next page: a one-component problem (HPT) solved

Numerical example



Project background

Maintenance of aircraft engines is expensive:

- *spare parts* cost up to 2 Mkr
- *total cost* for maintenance of a jet engine: 15–30 Mkr
- *rent* for a spare engine: 15 kkr/day

Opportunistic maintenance:

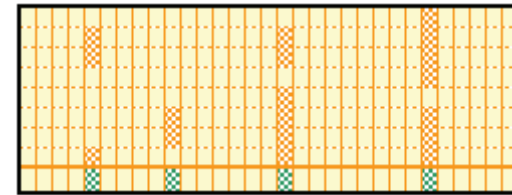
At each maintenance occasion, possible to *perform more maintenance than* what is absolutely *necessary*

⇒ totally fewer maintenance occasions

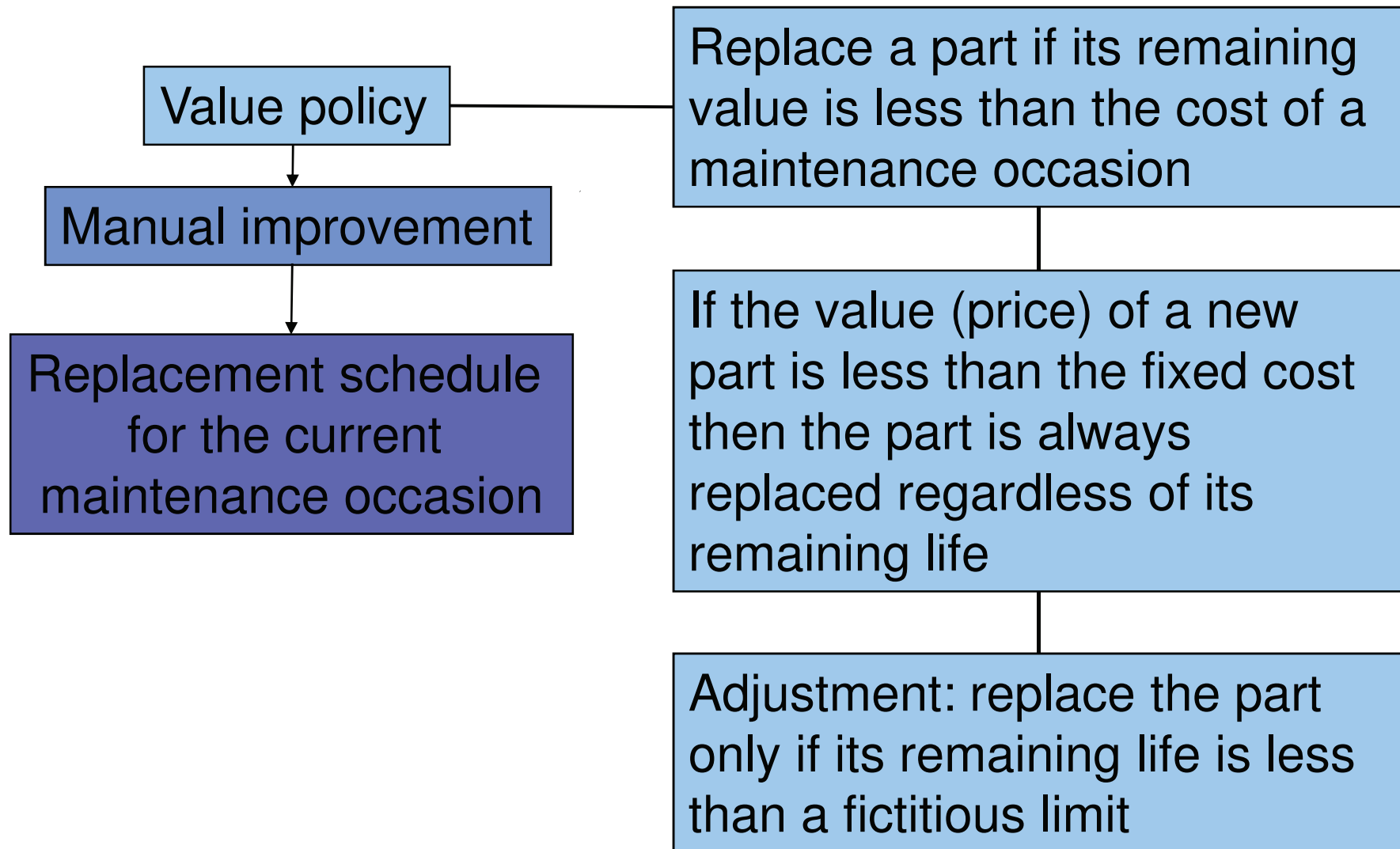
⇒ totally lower cost

The purpose of the project

- Create a *methodology* that generates good *replacement schedules* for components in aircraft engines
- Consider:
 - *Life time restricted* and "*on condition*"-components
 - *Fixed cost* when an engine/module is taken to the workshop
 - *Work costs* to set free engine modules their components
 - Utilize a *store* of used components
- *Minimize total flight hour cost* during the contract period



VAC:s existing value policy



A simple optimization model for the whole contract period

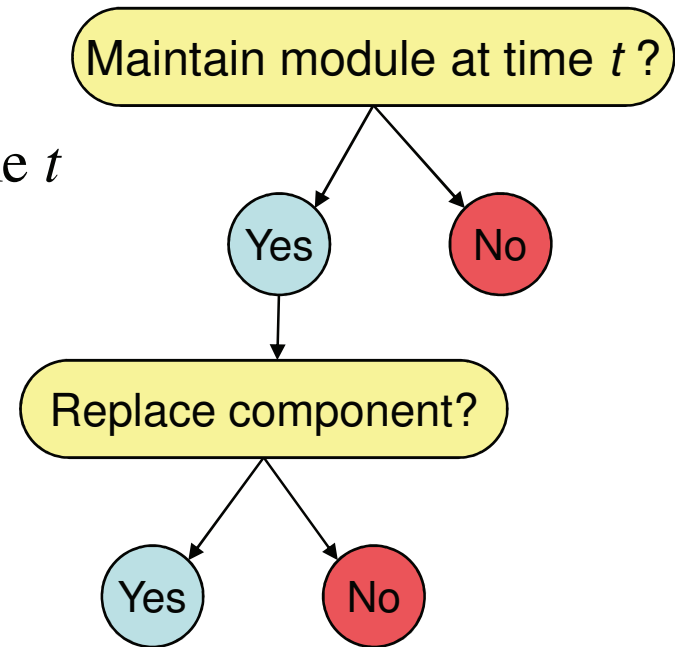
- For each component i in the module:
 - *Cost* of a new component: c_i
 - *Life* of a new component: T_i
 - *Remaining life* of current component: τ_i
- *Contract period* divided into T *time periods* $t = 1, \dots, T$
- Maintenance possible at start of each time period (*discrete time steps*)
- A *fixed cost* per maintenance occasion: d

A mathematical optimization model for maintenance planning

Definition of variables

$$z_t = \begin{cases} 1 & \text{if the module is maintained at time } t \\ 0 & \text{otherwise} \end{cases}$$

$$x_{it} = \begin{cases} 1 & \text{if component } i \text{ is replaced at time } t \\ 0 & \text{otherwise} \end{cases}$$



Basic mathematical model: one module, N parts, T time steps

$$\text{minimize } \sum_{t=1}^T \left(\sum_{i \in N} c_i x_{it} + dz_t \right)$$

$$\text{subject to } \sum_{t=1}^{\tau_i} x_{it} \geq 1, \quad i \in N, \quad \text{replace part before its remaining life is over}$$

$$\sum_{t=l}^{T_i+l-1} x_{it} \geq 1, \quad l=1, \dots, T-T_i+1, \quad i \in N, \quad \text{replace part at least once in a lifetime}$$

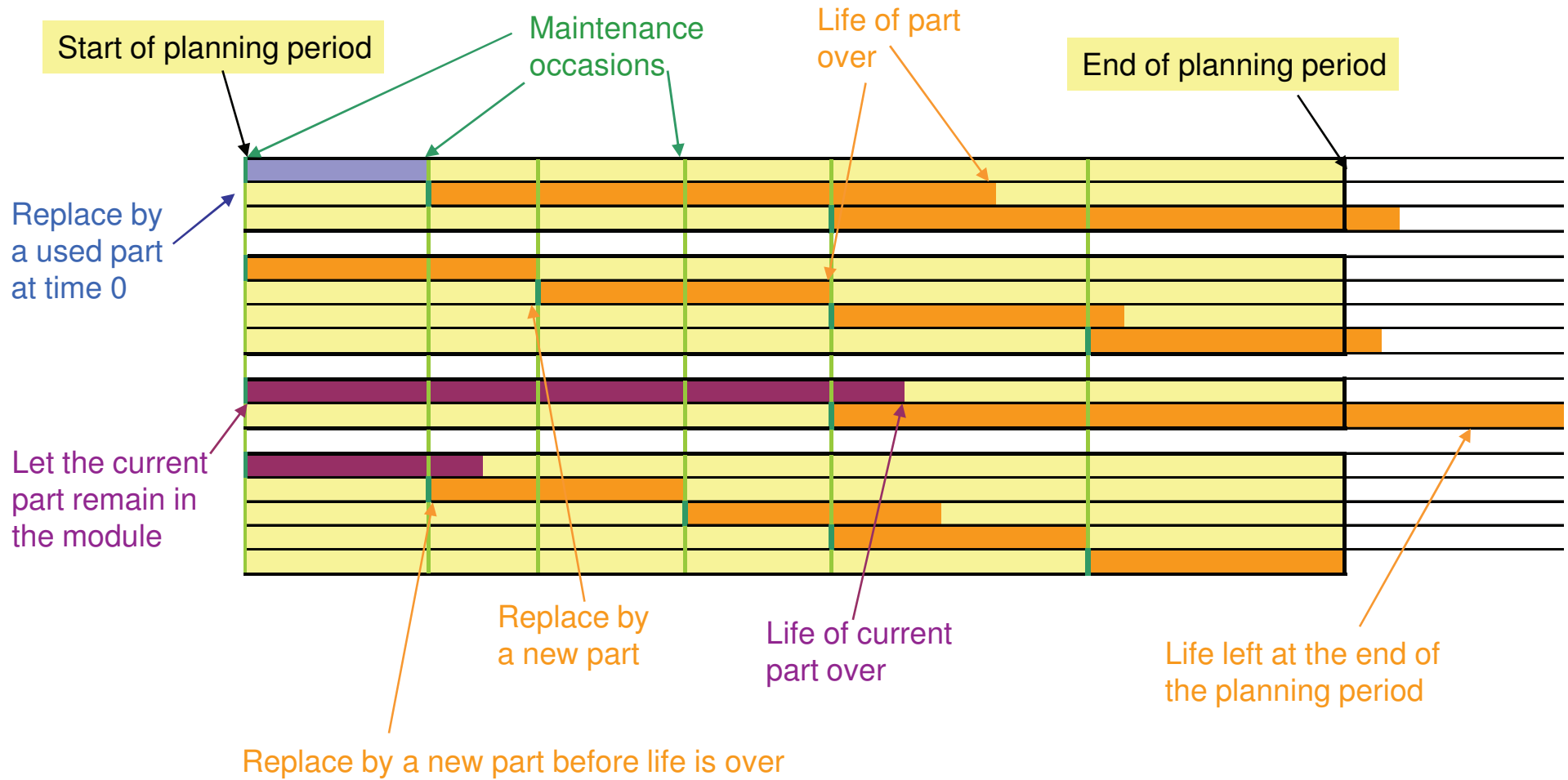
$$x_{it} \leq z_t, \quad t=1, \dots, T, \quad i \in N, \quad \text{replace part only at maintenance occasion}$$

$$x_{it} \in \{0, 1\}, \quad t=1, \dots, T, \quad i \in N,$$

$$z_t \in \{0, 1\}, \quad t=1, \dots, T.$$

- $x_{it} \in \{0, 1\}$ can be relaxed to $x_{it} \geq 0$ integrality property

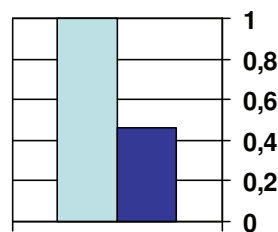
A maintenance schedule for four components in an engine module



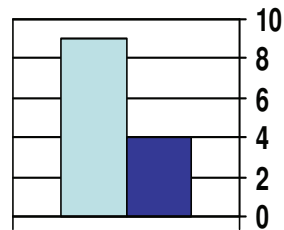
Comparison of the methods

- An engine module with 10 components
- Only life time restricted (deterministic) components

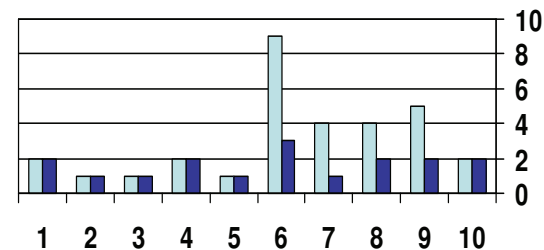
■ Value policy
■ Optimization



Total cost



maintenance occasions

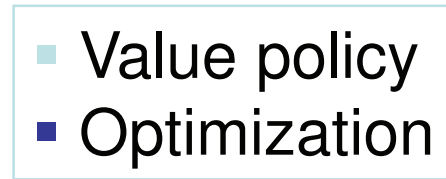


parts replaced

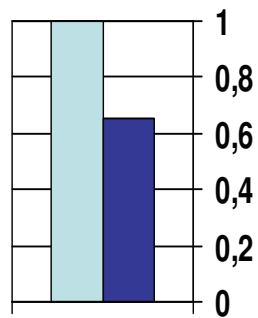
Comparison of the methods using stochastic simulations

- An engine module with 10 components
- Parts 1, 4, 5, 6, 9, 10 are OC (Weibull)

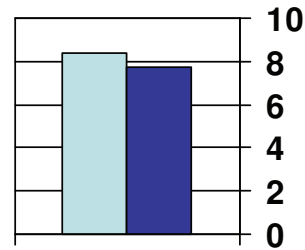
Part no	1	4	5	6	9	10
β	2	2	4	4	6	6



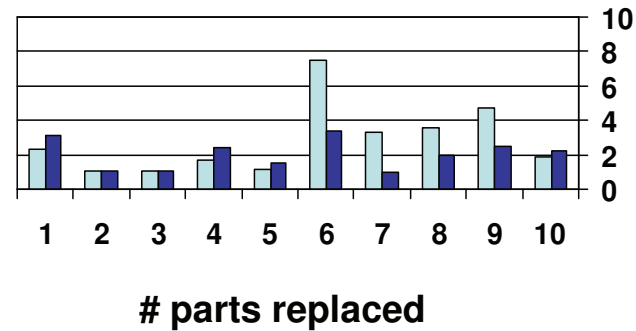
- Average values from 200 scenarios



Total cost



occasions



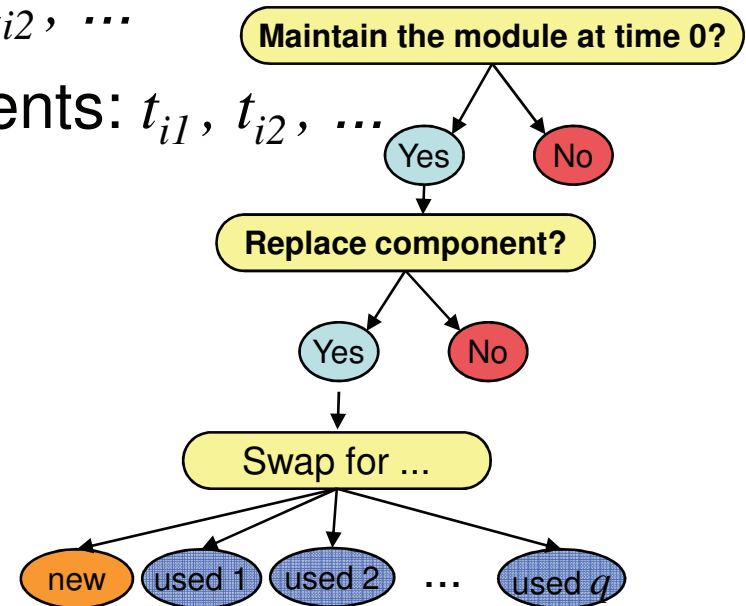
parts replaced

A store of used components

- For each part i in the module there is a *store of used components* at time 0 (at present maintenance occasion):
 - Costs* for used components: k_{i1}, k_{i2}, \dots
 - Remaining lives* of used components: t_{i1}, t_{i2}, \dots

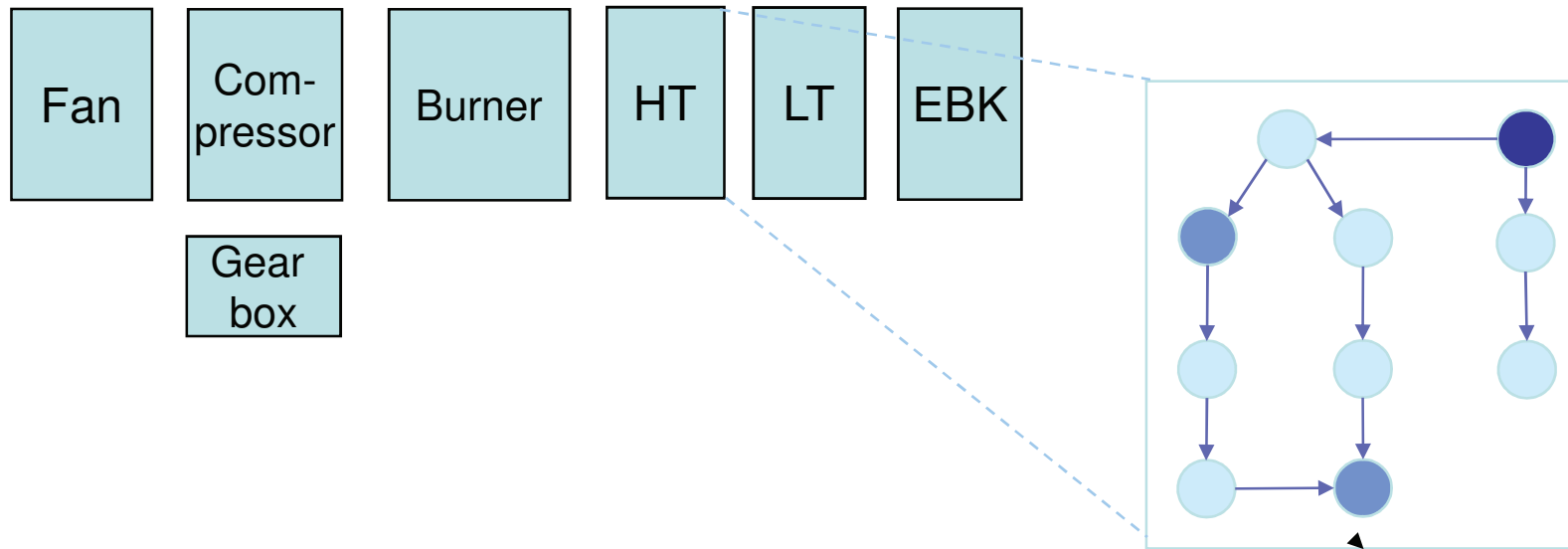
- Additional variables:

$$s_{ij} = \begin{cases} 1 & \text{if used individual } j \text{ of component } i \\ & \text{from the store is used at time 0} \\ 0 & \text{otherwise} \end{cases}$$



Several modules in an engine

- *Work costs to set modules free*
- *Work costs to set components free*



A mathematical model for a whole engine parameters

c_{it}^m = price of a spare of part i in module m at time t

\tilde{c}_{ik}^m = price for used individual k of part i in module m at $t = 0$

a_{it}^m = cost of removing part i in module m at time t

b_{nt} = cost of performing activity n at time t

d_t = fixed cost for maintaining the engine at time t

T = length of planning period (# time steps)

T_i^m = life of new part i in module m

\tilde{T}_i^m = remaining life of part i in currently in module m

e^{mi} = # used individuals of part i , module m in store at $t = 0$

\bar{T}_{ik}^m = remain. life of used indiv. k , part i , module m in store, $t = 0$

f_m = 1 if maint. of module m should be planned, = 0 if not

A mathematical model for a whole engine

variables

$x_{it}^m = 1$ if part i in module m is replaced at time t , $= 0$ if not

$u_{ik}^m = 1$ if part i in module m is replaced by used individual k at $t = 0$, $= 0$ if not

$y_{it}^m = 1$ if part i in module m is removed at time t , $= 0$ if not

$z_t^m = 1$ if module m is maintained at time t , $= 0$ if not

$v_{nt} = 1$ if activity n is performed at time t , $= 0$ if not

$w_t = 1$ if the engine is maintained at time t , $= 0$ if not

A mathematical model for a whole engine

$$\text{minimize } e \sum_{t=0}^{T-1} \left(\sum_{m \in M} \left(\sum_{i \in N^m} (c_{it}^m x_{it}^m + a_{it}^m y_{it}^m) \right) + d_t w_t + \sum_{n \in A} b_n v_{nt} \right) + \sum_{m \in M} \sum_{i \in N^m} \sum_{k=1}^{e^m} c_{ik}^m u_{ik}^m$$

$$\text{subject to } \phi^m \leq f_m, \quad t = 0, \dots, T-1, m \in M, \quad (1)$$

$$\sum_{t=0}^{\tilde{T}_i^m} x_{it}^m + \sum_{k=1}^{e^m} u_{ik}^m \geq f_m, \quad \text{if } (\tilde{T}_i^m \leq T-1) \quad i \in N^m, m \in M, \quad (2)$$

$$\sum_{t=l}^{T_i^m+l-1} x_{it}^m \geq f_m, \quad l = 1, \dots, T - T_i^m, i \in N^m, m \in M, \quad (3)$$

$$\sum_{t=0}^{\bar{T}_{ik}^m} x_{it}^m \geq f_m u_{ik}^m, \quad k = 1, \dots, e^m, i \in N^m, m \in M, \quad (4)$$

$$x_{it}^m \leq y_{it}^m, \quad i \in N^m, m \in M, t = 1, \dots, T, \quad (5)$$

$$u_{ik}^m \leq y_{ik}^m, \quad i \in N^m, m \in M, k = 1, \dots, e^m, \quad (6)$$

Mathematical model, continued

$$u_{i k}^m \leq z_0^m \quad i \in N^m, m \in M, k = 1, \dots, e^m, \quad (7)$$

$$y_{i t}^m \leq z_t^m, \quad i \in N^m, m \in M, t = 1, \dots, T, \quad (8)$$

$$\sum_{j \in \delta^m(i)} y_{j t}^m \geq y_{i t}^m, \quad i \in N^m, m \in M, t = 1, \dots, T, \quad (9)$$

$$z_t^m \leq \sum_{n \in A^m} v_n \quad m \in M, t = 1, \dots, T, \quad (10)$$

$$v_n \leq \sum_{n' \in A(n)} v_{n'} \quad n \in A, t = 1, \dots, T, \quad (11)$$

$$z_t^m \leq w_t \quad m \in M, t = 0, \dots, T-1, \quad (12)$$

$$u_{i k}^m, y_{i t}^m, z_t^m, v_n \in \{0,1\}, \quad i \in N^m, m \in M, n \in A, t = 0, \dots, T-1, k = 1, \dots, e^m, \quad (13)$$

$$x_{i t}^m, w_t \geq 0, \quad i \in N^m, m \in M, t = 0, \dots, T-1. \quad (14)$$

Tests and results

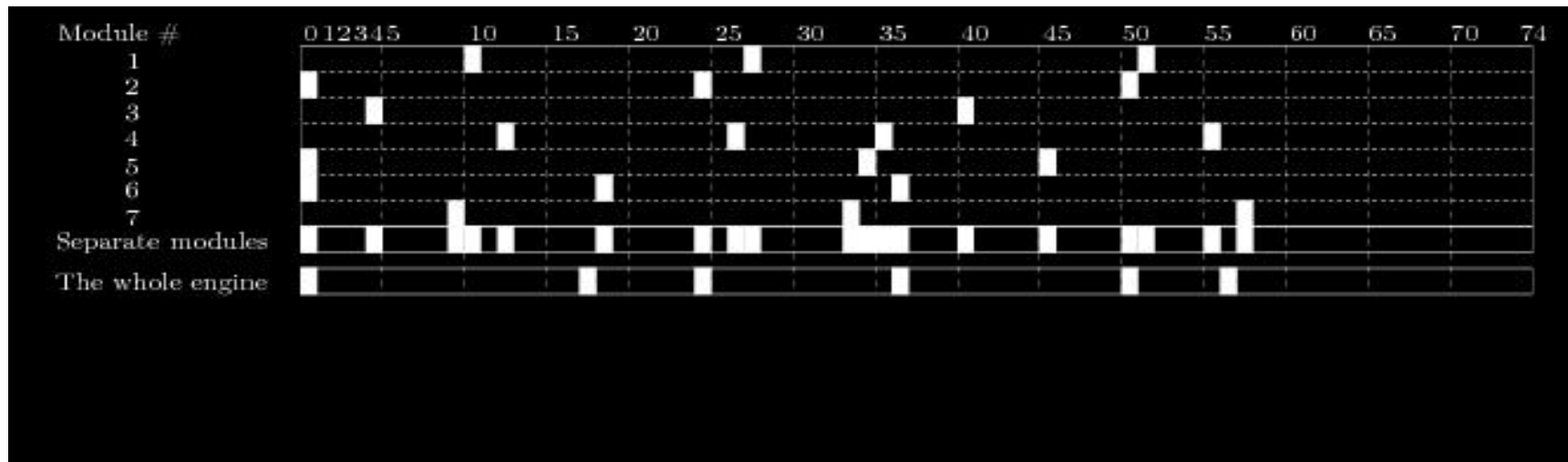
- Discretization: 33,33 flight hours per time step
- Length of the planning period = 2500 flight hours, $T=75$
- Total number of parts in the engine = 61
- Number of modules in the engine = 7
- Number of variables in the model = 10425
- Integrality property for some of the variables
- Number of binary variables in the model = 5775

$$\left(2^{5775} \approx 2.8 \cdot 10^{1738} \right)$$

Advantage of simultaneous optimization

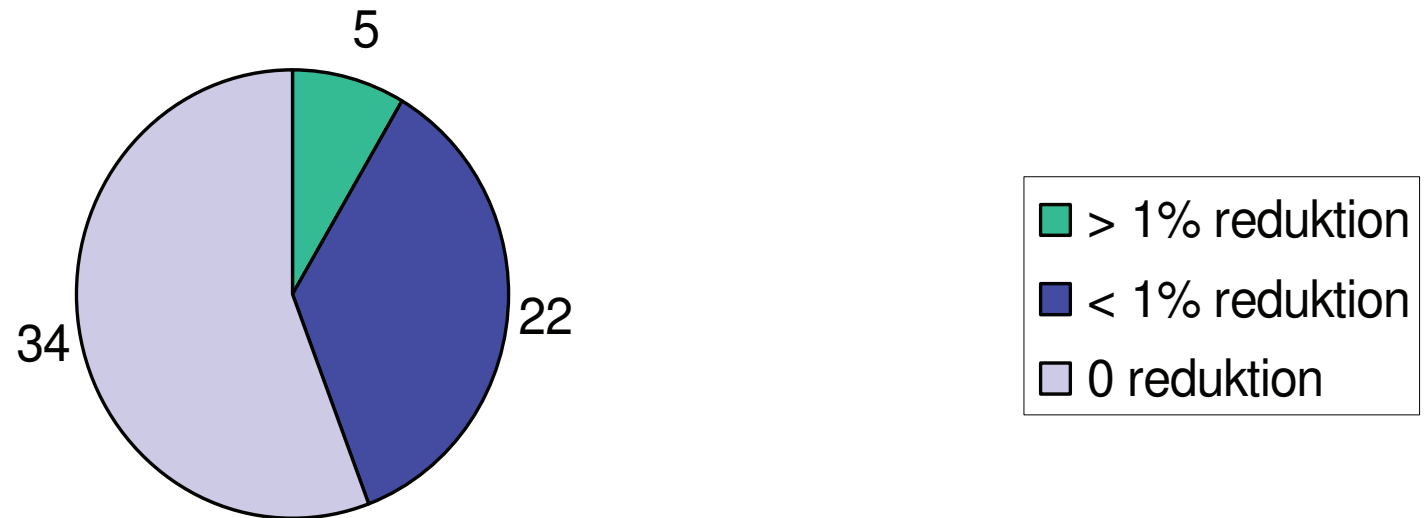
An old engine with a store of used spares at $t=0$

Optimization over:	# maintenance occasions	# replaced parts	Total cost (normalized)	CPU time (sec)
separate modules	19	90	1.222	3.08
the whole engine	6	92	1.000	1.25

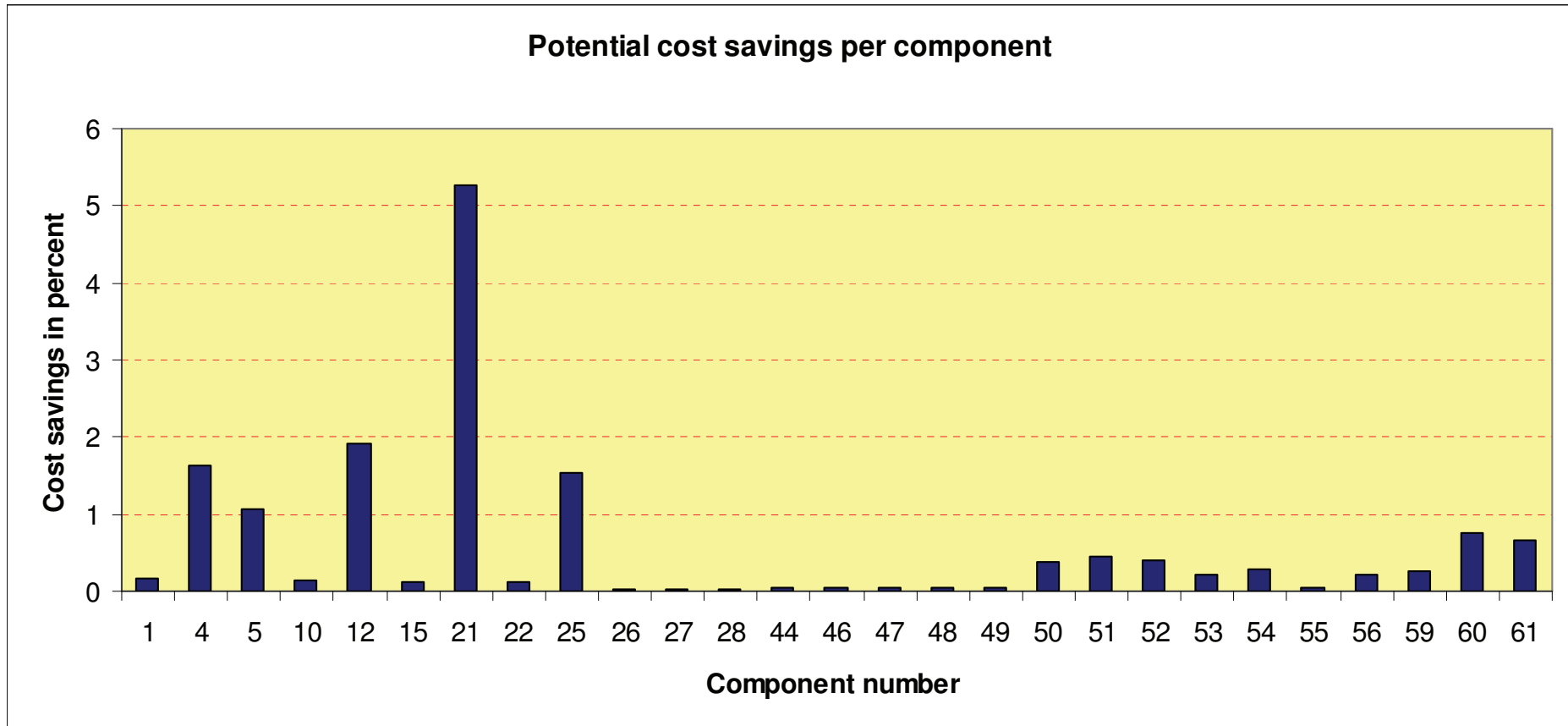


Product development

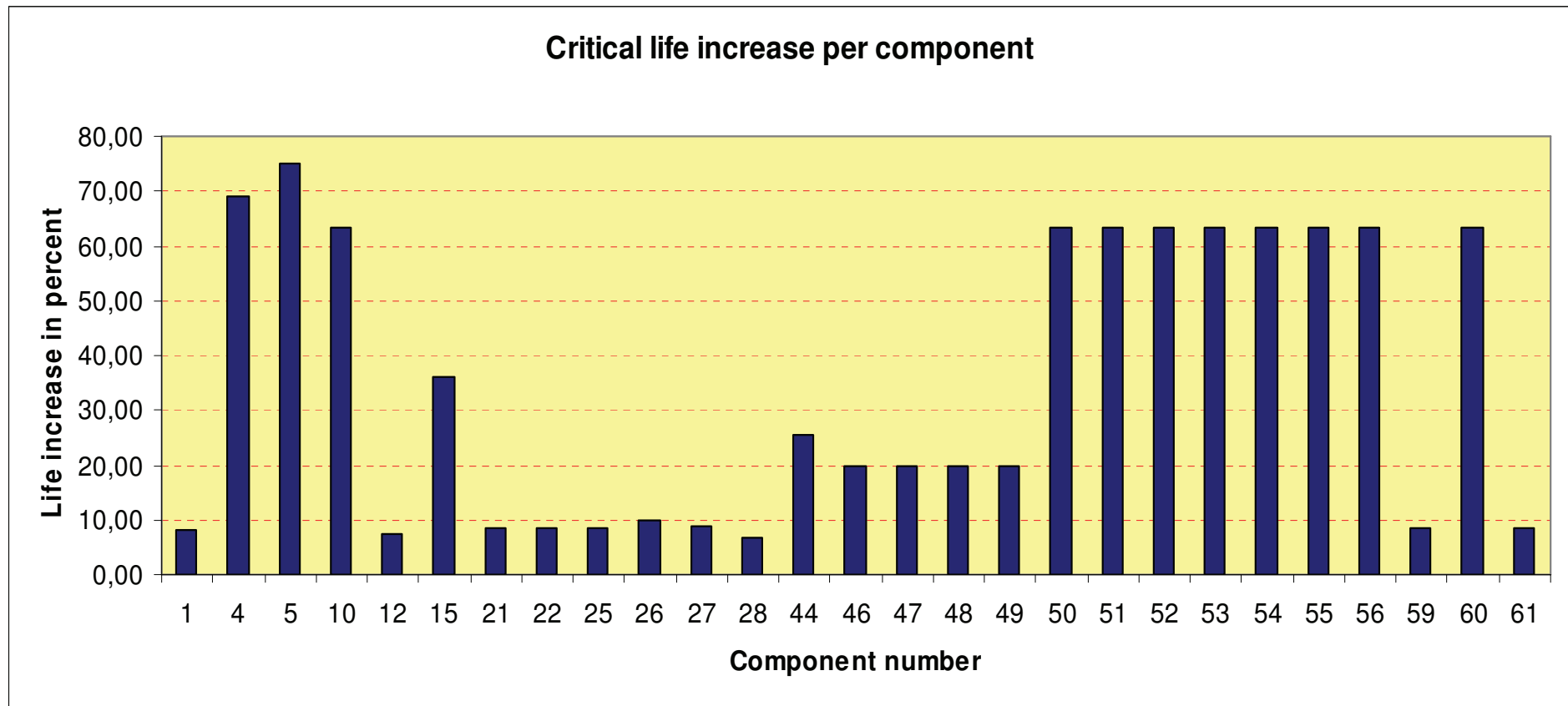
Proportion of components with a potential for reducing the total maintenance cost



Product development, continued

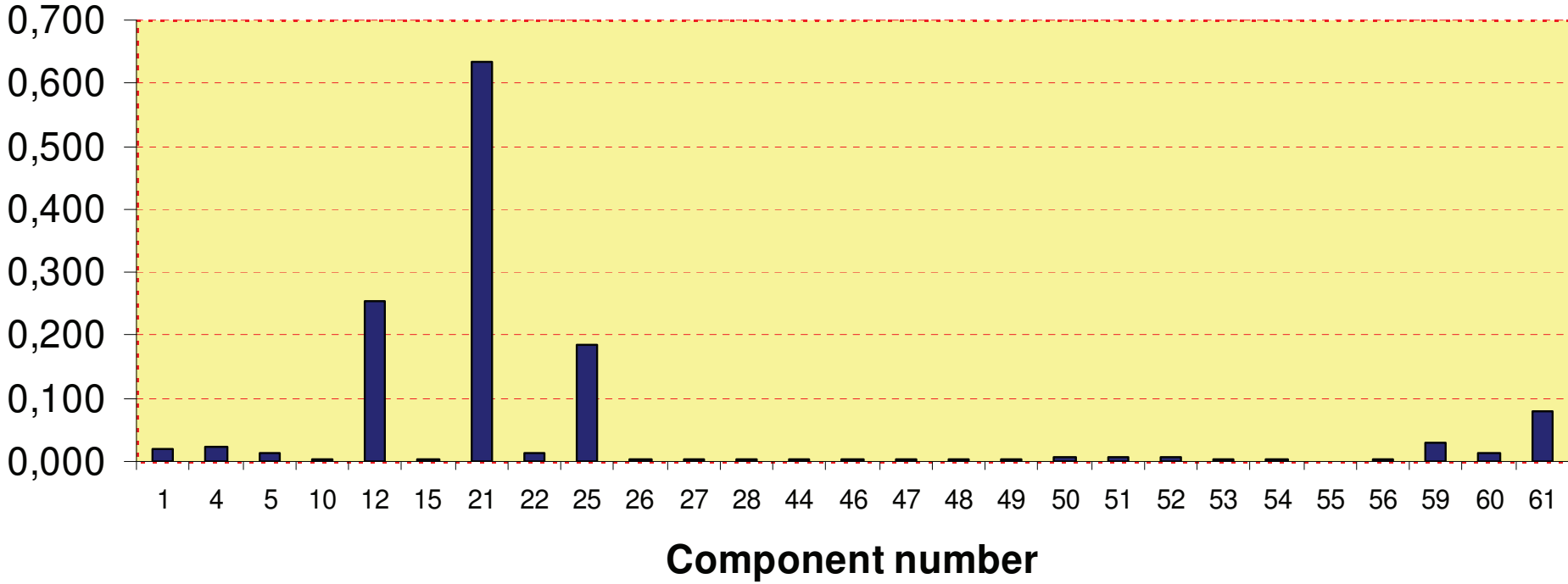


Product development, continued



Product development, continued

Cost reduction per percent life increase



Results on the Volvo Aero problems

- An individual engine module with 7 components: cost reduction 35%; reduction of maint. occasions 7%
- Complete engine of 10 modules (61 parts): cost reduction compared to maintaining them optimally but individually: 12%; reduction of maint. occasions 60%
- Product development: found 5 components that can potentially reduce maintenance costs more than 5% through prolonged lives

And the winner is ...

- The VAC project received the “Stora Produktivetspriset” at the 2010 Maintenance fair in Göteborg



Recent research and future plans

- Volvo Aero: maintenance of civil aircraft
- KTH/Elektro, Vattenfall, etc.: maintenance of nuclear power plants and wind power farms (data from Forsmark, resp. Lillgrund); major reduction in costs as well as in loss of production when utilizing meteorological data
- Modelling developments: uncertainties of lives (stochastic (programming) modelling); collaborations with companies that measure the status of components in order to improve life predictions