Maintenance optimization

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2011-04-11

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Maintenance optimization — a background

- Invitation 2000 from Volvo Aero Corporation (VAC): maintenance of the RM12 jet engine (JAS 39 Gripen)
- Paired PhD project between applied math/optimization and math statistics/material fatigue and reliability
- Optimization student: a model for opportunistic maintenance; superior to simpler policies
- Math statistics student: models for the determination of life distributions based on crack growth
- Continuation projects: VAC; maintenance of components in wind and nuclear power plants

A conversation with Bo Hägg, CEO Underhållsföretagen

- Maintenance = selling reliability at the least cost
- Maint. costs/year: 14K Billion SEK (EU), 275 Billion SEK (S)
- Maintenance is often seen merely as a cost
- Maintenance is sometimes done too often — inspections and measurements damage the systems; sometimes, like with road/rail infrastructure and the "Miljonprogramhusen" it is performed seldomly
- Truth: well performed maintenance is an investment in availability and safety

Maintenance principles

- Preventive maintenance: actions that prevent failure
- Corrective maintenance: actions after failure, repairs
- Condition based maintenance: measurements → predictions → actions according to a maintenance principle
- Opportunistic maintenance: when maintenance must be performed, also perform some preventive maintenance actions

A simple example, I

- System with n parts
- Life of part i : T_i time units (intervals)
- Horizon: T time units (ex. contract period)
- Cost of part i at time t: c_{it} monetary units
- Cost for performing any maintenance at time t: d_t monetary units

A simple example, II

- Variables are logical – do something or not
- Model uses binary variables:

$$x_t = \begin{cases} 1, & \text{if “something” is done at time } t \\ 0, & \text{otherwise} \end{cases}$$

- A decision often implies other necessary decisions
- Example: if part i shall be replaced at time t maintenance must be performed
- Such logical relations are equivalent to linear constraints:

$$\text{if A then B} \quad \Longleftrightarrow \quad x_A \leq x_B$$

The basic replacement problem, I

- Goal: minimize the total cost for a working system during the contract period:

Mathematical model

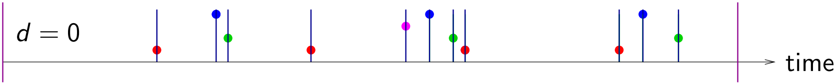
$$
\begin{aligned}
\underset{(x,z)}{\text{minimize}} \quad & \sum_{t=1}^{T} \left(\sum_{i=1}^{N} c_{it} x_{it} + d_t z_t \right), \\
\text{subject to} \quad & \sum_{t=l+1}^{l+T_i} x_{it} \geq 1, && l = 0, \ldots, T - T_i, \quad i = 1, \ldots, N, \\
& x_{it} \leq z_t, && t = 1, \ldots, T, \quad i = 1, \ldots, N, \\
& x_{it} \geq 0, && t = 1, \ldots, T, \quad i = 1, \ldots, N, \\
& z_t \leq 1, && t = 1, \ldots, T, \\
& x_{it}, z_t \in \{0, 1\}, && t = 1, \ldots, T, \quad i = 1, \ldots, N
\end{aligned}
$$

The basic replacement problem, II

- The objective is to minimize the total cost of having a working system during the contract period
- The first constraint states that, for any given item i in the system, during any time interval T_i time steps long, the part must be replaced at some point
- The second constraint ensure that we cannot perform the above replacement without paying the fixed cost d_t for performing a maintenance operation; once we do pay, any maintenance action becomes possible at that time
- The remaining constraints ensures that the variables only take meaningful values

Opportunistic maintenance or not?

- Example: four parts with different prices and lives
- A replacement is marked with a dot; its colour represents the type of part replaced



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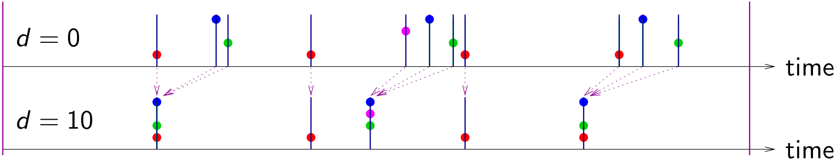
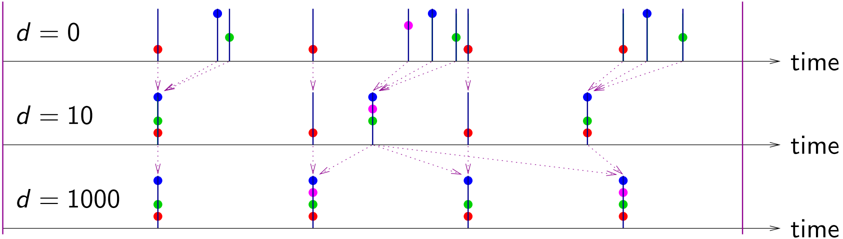


Opportunistic maintenance or not?

- Example: four parts with different prices and lives
- A replacement is marked with a dot; its colour represents the type of part replaced
- The bigger the fixed cost, the more interesting opportunistic maintenance becomes; also more items are replaced



Constraint structure—example

- Component 3: $\sum_{t=\ell+1}^{\ell+T_3} x_{3t} \geq 1,\ \ell = 0, \ldots, T - T_3$
- $T = 8,\ T_3 = 4 \quad \Longrightarrow \quad \sum_{t=\ell+1}^{\ell+4} x_{3t} \geq 1,\ \ell = 0, \ldots, 4$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_{31} \\ x_{32} \\ \vdots \\ x_{38} \end{bmatrix} \geq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Property I: the replacement problem is NP-hard

Theorem

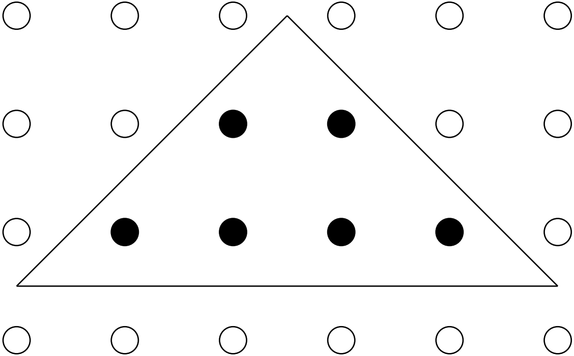
Set covering is polynomially reducible to the replacement problem

Property II: with fixed z the problem over x is easy

- The constraint matrix has the "consecutive ones" property, hence the problem over x can (for fixed z) be solved as an LP
- If for each i the component costs c_{it} increase (decrease) with t then we postpone (perform earlier) all replacements at maintenance occasions $\Longrightarrow$ greedy works!
- Under investigation: may be possible to find optimal LP basis in a greedy fashion *always*

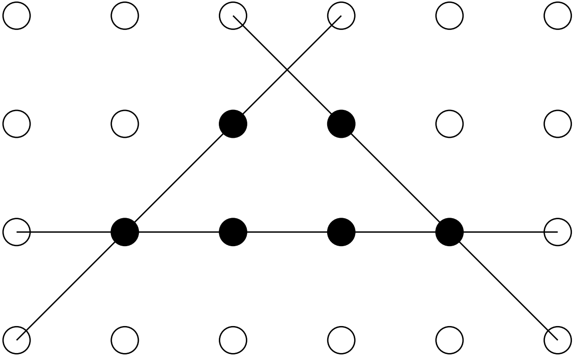
Property III: all inequalities are facet defining

No inequalities are facet defining



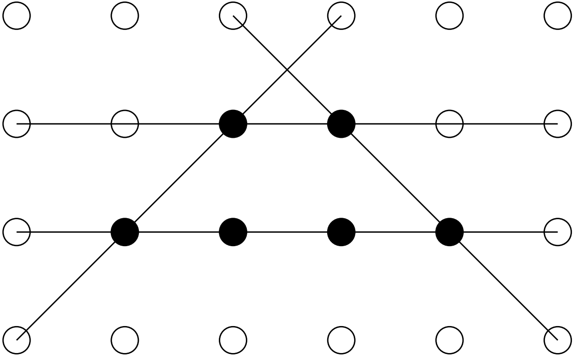
Property III: all inequalities are facet defining

All inequalities are facet defining



Property III: all inequalities are facet defining

Integral polyhederon



Additional facial structure, I

- Small-scale problems from practice can still take 30 hours using CPLEX 12.1
- Cutting planes, reformulations, heuristics?
- Proposal: add further structures implied by the original constraints
- Chvátal–Gomory inequalities, 1: consider $\sum_{i=1}^{m} a_{ij} x_j \geq b_i$, $i = 1, \dots, m$
- Chvátal–Gomory inequalities, 2: for $\mathbf{u} \geq \mathbf{0}^m$, take

$$\sum_{j=1}^{n} \left\lceil \sum_{i=1}^{m} u_i a_{ij} \right\rceil x_j \geq \left\lceil \sum_{i=1}^{m} u_i b_i \right\rceil$$

Additional facial structure, II

- If repeated enough, *all* additional facets can be generated
- In fact the above still holds with $u_i \in \{0, \frac{1}{2}\}$
- Work is currently performed to study how to incorporate these "cuts" within a branch & bound scheme (violated such cuts are added at crucial nodes in the tree)
- There exist software (e.g., GuRoBi) where such ideas can be tested

On the Volvo Aero project

- Aircraft engines are expensive:
  - Spare parts cost up to 2 MSEK
  - Total cost of maintenance of one engine: 15–30 MSEK
  - Maximizing "time on wing" is important, both for civil and military aircraft
- The aircraft engine of the JAS 39 Gripen aircraft consists of 7 modules and 61 parts in total
- A mathematical model has been constructed such that the entire engine maintenance is modelled, including work costs for disassembling the necessary modules and parts for each maintenance occasion
- This model has, in a discretized setting, slightly less than 6000 binary variables

Results on the Volvo Aero problems

- An individual engine module with 10 components: cost reduction 35%; reduction of # maint. occasions 7% (compared with a simple policy similar to that used at VAC)
- Complete engine of 7 modules (61 parts): cost reduction compared to maintaining them optimally but individually: 12%; reduction of maint. occasions 60%
- Product development: found 5 components that can potentially reduce maintenance costs more than 5% through prolonged lives

And the winner is . . .

- The VAC project received the “Stora Produktivitetspriset” at the 2010 Maintenance fair in Göteborg

Recent research and future plans

- Electrical Engineering: maintenance of wind power farms (data from Lillgrund); major reduction in costs as well as in loss of production when utilizing meteorological data
- Recruitments: a new PhD student; also a postdoc under reqruitment from the Energy area of strength, for work together will Electrical Engineering on wind power
- New area: maintenance of rails and wheels (next page)
- Modelling developments: uncertainties of lives (stochastic (programming) modelling); collaborations with companies that measure the status of components in order to improve life predictions
- Algorithmic developments based on facet generation

Maintenance of rails and wheels, I

- Paired PhD project in collaboration with CHARMEC
- Background: increased wear of rails and wheels due to an increase in speeds and loads
- Aim to develop decision support tools for the optimization of inspection and maintenance of rails and wheels wrt. LCC, safety and maintenance logistics
- The other PhD student (at CHARMEC) models the progressive degradation of rails and wheels
- Will result in advanced knowledge on how component condition indicators can be efficiently used in an optimization

Maintenance of rails and wheels, II

- In the picture below we see how a part of a rail is degraded over time (measured in portions of the rail having crack lengths in given intervals)
- As shown in the figure, several levels of maintenance can be performed, with different effects (and different costs)
- The optimization will determine which maintenance action is the most appropriate at any given time