Assignment 1: Energy system modelling

Given below is a mathematical model for minimizing the costs of meeting climate targets. An implementation of the model in AMPL can be found on the course homepage: `www.math.chalmers.se/Math/Grundutb/CTH/mve165/1112/` To pass the assignment you should (in groups of two persons) give satisfactory answers to the following questions in a written report. The report should be 2–3 pages long excluding illustrating diagrams and it should be

submitted in PingPong at latest Friday 30 of March 2012 at 17.00.

In each of the eight questions below the variation of data and/or constraints shall be made starting from the original data (resurs-data.dat) and model (resurs-modell.mod), respectively. Use the case of 450 ppm (see the file resurs-data.dat) as the basic scenario for the maximum allowed value of the yearly amount of emissions.

Questions

1. (a) What is the global discounted cost for the energy system if there is no climate policy in place? Compare to the cost of stabalizing the atmospheric carbon content of 400, 450 and 500 ppm.

   (b) What is the abatement cost for these scenarios (the difference between the total cost of a stabilization scenario and of the baseline[1] scenario)?

2. (a) Compare the total use of oil and coal during all the time steps of the 21st century for the baseline scenario and the three stabilization scenarios. Draw illustrating graphs and explain the differences.

   (b) Find out how much these reserves can be reduced while staying feasible. Compare the accumulated carbon dioxide emissions to the original baseline scenario. Draw illustrating graphs and explain the differences. What implication does the depletion of the oil reserves have on the severity of the climate change problem?

   (c) Try to find out and discuss how the global energy system would react on an infeasible scenario, i.e., if the oil and/or coal reserves are too small, and how this could be incorporated in the mathematical model.

3. Compare the energy supply for residential heat, industrial heat, electricity, and transport for the baseline and for a 450 ppm scenario. Draw

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[1]"Baseline" means "no bounds on the amount of emissions".

appropriate illustrating graphs. In which sectors does the carbon dioxide abatement take place first? Explain why.

4. Increase the fraction of solar that can be used for residential heat to 0.8 and reduce the possibility of using biomass for industrial heat to 0.2. How does the biomass usage in the transport sector change? Draw illustrating graphs and compare for all time steps over the century. Explain why the use of biomass in the transport sector changes.

5. (a) What is the cost per EJ/year for *reducing* the potential for energy supply from biomass fuel (inflate the numbers for each time step with the discount rate r)?
   (b) For how many units of the reduction of the yearly potential P does this hold? What happens then? Why?
   Draw illustrating diagrams (over all the time steps) of the different results.

6. What are the *reduced costs* for extracting fuels and for the investment in equipment for converting energy from solar and biomass to electricity (inflate the numbers for each time step with the discount rate r)? What does this imply? Draw illustrating graphs and relate the reduced costs to the investments made in the respective time steps.

7. Increase the potential use of nuclear power from 20% of the electricity demand to 90%.
   (a) What would be the impact—with respect to energy supply from different sources—on the transport system?
   (b) Relate this to the changes of the *shadow price* for carbon dioxide emissions (inflate the numbers for each time step with the discount rate r).

   Compare for all time steps over the century. Draw illustrating diagrams of the different results.

8. Increase the investment costs for converting energy from $solar_{H2}$ to all types of output energy by 100%.
   (a) What is the effect on the sources for energy supply?
   (b) What is the effect on the shadow prices (in the respective time steps) for emissions (inflate the numbers for each time step with the discount rate r)?

   Draw illustrating graphs and compare for all time steps over the century. Discuss and (try to) explain.

9. Suppose that it is be concluded that the known energy resouces will not be enough for the global demand. How can a (tentative) new energy source—with a very high development cost—be incorporated in the optimization model?

The mathematical model

Sets

- The set of energy inputs (in AMPL: `ENERGY_IN`):
  $\mathcal{X} = \{\text{solar}_{\text{H2}}, \text{oil}, \text{coal}, \text{solar}, \text{bio}, \text{nuc}, \text{elec}, \text{res}, \text{ind}, \text{transp}\}$.

- The set of energy outputs (in AMPL: `ENERGY_OUT`):
  $\mathcal{Y} = \{\text{elec}, \text{res}, \text{ind}, \text{transp}\}$, where "elec"=electricity,
  "res"=residential heat, "ind"=industrial heat, "transp"=transport.

- The set of fuels (in AMPL: `FUELS`): $\mathcal{F} = \{\text{oil}, \text{coal}, \text{bio}, \text{nuc}\}$.

- The set of time steps: $\mathcal{T} = \{1, \dots, 12\}$ corresponding to the years[2]
  $\{2010, \dots, 2100, 2110, 2120\}$ (the index 0 represents year 2000)
  (in AMPL: `YEARS`).

Parameters

- D_{yt} = yearly demand of energy output y in time step t, $y \in \mathcal{Y}$, $t \in \mathcal{T}$
  [EJ/year] (in AMPL: `dem[eo,t]`).

- δ_{xy} = maximum allowed value for energy conversion per year [EJ/year].

- Q_{xy} = usage of energy the year 2010 (amount of energy converted from
  type x to type y year 2010) [EJ/year] (in AMPL: `init_energy[ei,eo]`).

- P = yearly potential for energy supply from biomass fuel [EJ/year]
  (in AMPL: `bio_pot`).

- R_f = stock of fossil fuel of type f (EJ) (in AMPL: `supply_pot`).

- η_{xy} = efficiency of converting energy from type x to type y, $x \in \mathcal{X}$, $y \in \mathcal{Y}$
  (in AMPL: `effic[ei,eo]`).

- ℓ_{xy} = load factor (usage level) of the capital stock per year for converting
  energy from type x to type y (in AMPL: `lf[ei,eo]`).

- γ_{xyt} = maximum allowed capital investment in equipment for energy
  conversion from type x to type y in time step t [TW/year].

- α_{xy} = factor constraining energy conversion with respect to demand
  (in AMPL: `limits`).

- τ = life of capital [years] (in AMPL: `life_plant`).

- r = discount rate (in AMPL: `r`).

- M = millions of seconds in one year ($= 31.536$)
  (in AMPL: `Msec_per_year`).

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[2]The time period studied is 2010–2100. In order to avoid errors due to boundary conditions, the time horizon for the computations is, however, extended to the year 2120.

- λ = length of time steps in years (= 10) (in AMPL: `t_step`).

- p_f = cost for extracting fuel of type f [GUSD/EJ] (in AMPL: `price[f]`).

- k_{xy} = investment cost for equipment for converting energy from type x to type y [GUSD/TW] (in AMPL: `cost_inv[ei,eo]`).

- v_x = capital cost for vehicles, energy type x [GUSD/EJ] (in AMPL: `veh_cost[ei]`).

- β_f = carbon emission from fossil fuel type f [Mton C/EJ] (in AMPL: `emis_fact[f]`).

- $\overline{\Omega}_t$ = maximum allowed value of the yearly amount of emissions in time step t [Gton C/year] (in AMPL: `emis_up[t]`).

Variables

- S_{ft} = yearly primary supply of fuel f in time step t [EJ/year] (in AMPL: `supply_1[f,t]`).

- E_{xyt} = the yearly amount of energy converted from type x to type y in of time step t [EJ/year] (in AMPL: `en_conv[ei,eo,t]`).

- e_{yt} = the yearly amount of electricity used as secondary input for output energy of type y in time step t [EJ/year] (in AMPL: `extra_dem[eo,t]`).

- g_{yt} = the yearly supply of input energy for output energy of type y in of time step t [EJ/year] (in AMPL: `en_supply[eo,t]`).

- C_{xyt} = yearly capital stock in time step t for energy conversion from type x to type y [TW] (in AMPL: `capital[ei,eo,t]`).

- I_{xyt} = the yearly capital investment in time step t in equipment for energy conversion from type x to type y [TW/year] (in AMPL: `cap_invest[ei,eo,t]`).

- U_t = yearly carbon emission in time step t [Gton C/year] (in AMPL: `C_emission[t]`).

- Π_t = the annual cost in time step t [GUSD/year] (in AMPL: `annual_cost[t]`).

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The mathematical model

$$\text{minimize} \quad \lambda \sum_{t \in \mathcal{T}} \left[(1+r)^{-(t-1)\lambda} \right] \Pi_t, \tag{1}$$

subject to

$$\sum_{f \in \mathcal{F}} p_f S_{ft} + \sum_{x \in \mathcal{X}} \left(\sum_{y \in \mathcal{Y}} k_{xy} I_{xyt} + v_x E_{x,\text{transp},t} \right) = \Pi_t, \qquad t \in \mathcal{T}, \tag{2}$$

$$S_{\text{bio},t} \leq P, \qquad t \in \mathcal{T}, \tag{3}$$

$$\lambda \sum_{t \in \mathcal{T}} S_{ft} \leq R_f, \qquad f \in \{\text{oil}, \text{coal}, \text{nuc}\}, \tag{4}$$

$$\sum_{y \in \mathcal{Y}} E_{fyt} = S_{ft}, \qquad f \in \mathcal{F}, t \in \mathcal{T}, \tag{5}$$

$$\sum_{y \in \mathcal{Y}} E_{\text{elec},y,t} = e_{\text{elec},t}, \qquad t \in \mathcal{T}, \tag{6}$$

$$\sum_{x \in \mathcal{X}} \eta_{xy} E_{xyt} - e_{yt} = D_{yt}, \qquad y \in \mathcal{Y}, t \in \mathcal{T}, \tag{7}$$

$$\sum_{x \in \mathcal{X}} E_{xyt} = g_{yt}, \qquad y \in \mathcal{Y}, t \in \mathcal{T}, \tag{8}$$

$$E_{xyt} - M \ell_{xy} C_{xyt} \leq 0, \qquad x \in \mathcal{X}, y \in \mathcal{Y}, t \in \mathcal{T}, \tag{9}$$

$$\lambda I_{xyt} + \left[\left(1 - \tfrac{1}{\tau}\right)^{\lambda} \right] C_{x,y,t-1} = C_{xyt}, \qquad x \in \mathcal{X}, y \in \mathcal{Y}, t \in \mathcal{T}, \tag{10}$$

$$C_{xy0} = 0, \qquad x \in \mathcal{X}, y \in \mathcal{Y}, \tag{11}$$

$$I_{xyt} \leq \gamma_{xyt}, \qquad x \in \mathcal{X}, y \in \mathcal{Y}, t \in \mathcal{T}, \tag{12}$$

$$E_{xy1} \leq Q_{xy}, \qquad x \in \mathcal{X}, y \in \mathcal{Y} \tag{13}$$

$$\eta_{xy} E_{xyt} - \alpha_{xy} e_{yt} \leq \alpha_{xy} D_{yt}, \qquad x \in \mathcal{X}, y \in \mathcal{Y}, t \in \mathcal{T}, \tag{14}$$

$$E_{xyt} \leq \delta_{xy}, \qquad x \in \mathcal{X}, y \in \mathcal{Y}, t \in \mathcal{T}, \tag{15}$$

$$\sum_{f \in \mathcal{F}} \beta_f S_{ft} = U_t, \qquad t \in \mathcal{T}, \tag{16}$$

$$U_t \leq \overline{\Omega}_t, \qquad t \in \mathcal{T}, \tag{17}$$

$$S_{ft}, E_{xyt}, e_{yt}, C_{xyt}, I_{xyt}, U_t, \Pi_t \geq 0, \qquad x \in \mathcal{X}, y \in \mathcal{Y}, f \in \mathcal{F}, t \in \mathcal{T}. \tag{18}$$

Explanation of the mathematical model

(1) Objective: minimize the sum over the years of discounted annual costs (in AMPL: `tot_cost_M`).

(2) The annual cost is composed by costs for extracting fuels, inverstment costs for energy conversion equipment, and (extra) vehicle costs for energy conversion (in AMPL: `annual_cost_Q`).

(3) The yearly primary supply of biomass fuel in time step t may not exceed the yearly supply potential (in AMPL: `supply_pot_Q`).

(4) For fossil fuels (oil, coal, and nuclear) the sum over the whole time period of the yearly primary supply of fuel may not exceed the stock of fossil fuel (in AMPL: `reserves_Q`).

(5) In each time step, t, the primary supply of fuel f is converted to (four) different energy output types, y (in AMPL: `supply_1_Q`).

(6) In each time step, t, the amount of electrical energy converted to output energy equals the amount of electricity used as secondary input (in AMPL: `supply_2_Q`).

(7) In each time step, t, and for each output type, y, the amount of energy per year converted to type y minus the amount of y used as secondary input equals the demand for y (in AMPL: `energy_demand_Q`).

(8) In each time step the yearly supply of input energy for otput energy type y equals the sum over the input energy types of energy converted (in AMPL: `en_supply_Q`).

(9) The yearly amount of energy converted from type x to type y in time step t many not exceed the effective (w.r.t. load factor) yearly capital stock (in AMPL: `capital_lim_Q`).

(10) For each input type, x, and each output type, y, of energy conversion, the yearly capital stock in time step t equals the discounted (w.r.t. life of capital) yearly capital stock in time step $t-1$ plus the yearly capital investment in time step t (in AMPL: `capital_Q`).

(11) For each input type, x, and each output type, y, of energy conversion, the capital stock in time step 0 (i.e., year 2000) equals 0 (in AMPL: `capital_init_Q`).

(12) The yearly capital stock is limited for certain time steps and types of input energy (in AMPL: `cap_invest_UB_1,...,cap_invest_UB_4`).

(13) The yearly amount of energy converted from input type x to output type y in time step 1 (i.e., year 2010) may not exceed the supply of energy in year 2010 (in AMPL: `q_init_energy`).

(14) The yearly amount of energy converted from input type x to output type y in time step t may not exceed a certain fraction of the yearly demand of output energy y. (in AMPL: `q_limits`).

(15) The yearly amount of energy converted from input type x to output type y in time step t may not exceed the maximum value for energy conversion per year—due to levels of technical development.

(16) The yearly amount of carbon emmision during time step t equals the sum of carbon emission per year from the primary supply of fuels (in AMPL: `emission_Q`).

(17) The amount of carbon emmision in time step t may not exceed the maximum allowed value (in AMPL: `emis_UB`).