Chalmers University of Technology
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Mathematical Sciences
Optimization
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MVE165
MMG630
Applied Optimization
Assignment information
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Assignment 3a: Storage, refinement and distribution of sugar

In an appended document is a description of the problem to determine the best distribution channel for sugar in different regions and market segments, and the problem to determine on multi-period transportation, refining, and storage of sugar. Attached is also data needed for the problem formulation.

The assignment tasks are to `(a)` formulate the problems in mixed integer linear programming models, `(b)` model and solve them using AMPL and CPLEX (or, e.g., Matlab and any MILP solver; see *Computer exercise* on the course homepage), and `(c)` analyze the results and answer a number of questions given below.

To pass the assignment you should (in groups of two persons) `(i)` write a **detailed report** that gives satisfactory answers and explanations to the questions. You shall also estimate the number of hours spent on this assignment and note this in your report.

The file containing your report shall be called **Name1-Name2-Ass3a.pdf**, where "Name$k$", $k = 1, 2$, is your respective family name. **Do not forget to write the authors' names also inside the report.**

The report should be

submitted in PingPong at latest Monday 7th of May 2012.

Your shall also `(ii)` present your assignment orally at a seminar on
10th, 11th or 15th of May 2012.

The seminars are scheduled via a doodle link, which will be published on the course home page. Presence is mandatory at at least one of these seminars.

Exercises to perform and questions to answer

1. Formulate a mixed integer linear programming model that solves the second-level problem (described in the document `SCDS-2011.pdf`) to determine which warehouses should be established, and on inventories, refinement, and distribution of sugar. Note that some of the input data to this problem comes from the solution to the first-level problem which is given in the data file.

2. Implement the model from 1. in AMPL and solve it using CPLEX. Present and interpret the results and your findings. Comment also on the CPU time required to solve the problem and compare with the time required to solve the continuous relaxation of the model.

3. Study the sensitivity of the model formulated in 1. with respect to the following entities:

   (a) The refining capacity of raw sugar in the internal refineries of SCDS (parameter P_t). Let the values of all these parameters vary between 40% and 160% of their current value at the same time. That is, $P_t^{\text{new}} = \varphi P_t^{\text{old}}$, where $\varphi \in [0.4, 1.6]$ and $t = 1, \ldots, 12$. Is it necessary to solve for "all" values of φ, or can you draw some conclusions from the results for just some values? Analyze the impact of these variations on the model.

   (b) Assume that the refining capacity of the external refineries 11–20 is no longer available, i.e., the index values $j = 11, \ldots, 20$ and $t = 1, \ldots, 12$. How does this influence the solution to the problem?

   (c) Vary the holding cost per month of refined and raw, respectively, sugar in the (internal) warehouses of SCDS in Khuzestan, i.e., the parameters H and $\overline{H}$. What is the influence on the solution? At what levels of the values of H and $\overline{H}$ do which interesting effects occur?

   (d) Vary the cost of refining raw sugar in the external refineries, i.e., the parameter B. How is the solution influenced? At what levels of the value of B do which interesting effects occur?

   (e) Vary the cost of loading/unloading of sugar from/to the trucks, i.e., the parameter L. What is the influence on the solution? At what levels of the value of L do which interesting effects occur?

SCDS, a holding group consisting of sugarcane farms, refineries and warehouses in south western Iranian province of Khuzestan, is one of the greatest providers of sugar products in Iran. They also have contracts with sugar beet refineries (which will be called ***external refineries*** from now on) all around the country to use their excess capacity for **refining raw sugar** as well as for **temporary storage of raw/refined sugar**. However, refined sugar cannot be kept in the warehouse of an external refinery for more than a month. Currently, SCDS distributes sugar in the market via 7 very big independent wholesalers, none of whom is an agent of SCDS. However, SCDS thinks they can have bigger profit margin, if they distribute the sugar themselves. There are 30 provinces in the country. Each province is assumed to have a known deterministic demand of (refined/white) sugar in 4 market segments: households, workshops and confectionaries, small industries and big industries. Different distribution methods can be employed instead of the mentioned wholesalers to distribute sugar in each market segment in each province including: distribution via syndicates, small agents or big agents and direct sale. The provinces are categorized within 7 regions with respect to similarities as shown in Figure 1. The same distribution method will be used for the customers of the same market segment in all of the provinces of the same region.

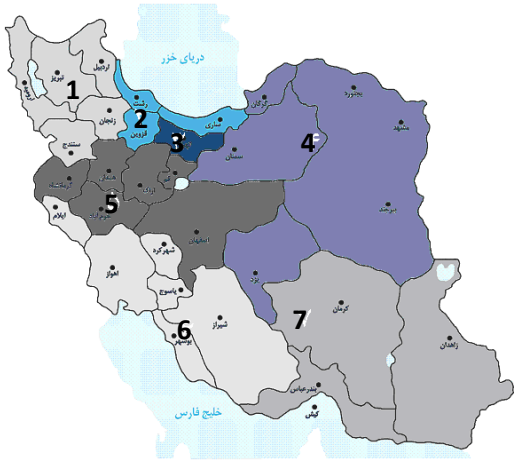

Figure 1. Provinces grouped into 7 regions

A qualitative method is employed to determine the best distribution methods for each region in each market segment. The results are summarized in Table 1.

Table 1. Chosen distribution methods for the market segments in the regions

Market segment → Region ↓	Households	Workshops	Small industries	Big industries
1) North-west	Small agents	Small agents	Direct sale	Big agents
2) North	Small agents	Small agents	Direct sale	Big agents
3) Tehran	Small agents	Small agents	Big agents	Big agents
4) North-east	Small agents	Small agents	Big agents	Big agents
5) Centre	Small agents	Via syndicates	Big agents	Big agents
6) South-west	Small agents	Small agents	Big agents	Big agents
7) South-east	Small agents	Small agents	Big agents	Big agents

Then, two optimization models in a 2-level hierarchy are employed which determine the storages points in the network and their corresponding capacities, to assign them to the customers and to give a plan for refining, storage and transportation of sugar. In this assignment, you only have to model and solve the second-level model. However, the first-level model and its corresponding outcomes which are used in the second-level model are briefly explained below.

The first-level model determines the number of agents in the provinces and assigns them to the customers. An agent can provide the needed sugar for the customers in only in its own region. Each small and big agent has got a maximum distribution capacity of 5000 and 20000 tons/year (distribution capacity is not the same as storage capacity). It is possible that a province receives the needed sugar, totally or partly, from the agents of the other provinces in the same region.

The following constraints must be considered in the first-level model:

- The demand assigned to the small and big agents in each province must be satisfied via the agents in the same region.
- Small and big agents located in each province cannot cover more than 120% and 150% (respectively) of the demand of their own province in total.
- The number of agents must be integer.

The objective of the model is to minimize the total transportation cost of sugar between agents and the assigned customers (in the centers of the provinces).

Note that it is assumed that the whole demand of a province is located in its center [of gravity]. This assumption also holds in the second-level model.

The decision variables of the first-level model are defined as follows:

- x_i : number of small agents in province i , $i = 1...30$
- x'_i : number of big agents in province i , $i = 1...30$
- y_{ij} : part of the demand assigned to small agents of the province i which is covered by the small agents of the province j , $i, j = 1...30$
- y'_{ij} : part of the demand assigned to big agents of the province i which is covered by the big agents of the province j , $i, j = 1...30$

However, the results show that the agents assigned to a province only cover the demand of the corresponding customers in the same province. That is, in the optimal solution $y_{ij} = y'_{ij} = 0$ if $i \neq j$.

Optimal number of agents in each province (x_i and x'_i) are given in the data file to be used in the second-level model.

In the second-level model, it is determined where should the intermediate warehouses be located (if any). There is a list of candidate locations for intermediate warehouses. The storage capacity of the intermediate warehouses must also be determined. An intermediate warehouse can be big, medium or small (the corresponding capacities are given later). Also, for each month of the year, it is determined how much sugar is refined in the internal and external refineries and how much sugar is stored in the warehouses of the SCDS in Khuzestan (internal warehouses), external refineries and intermediate warehouses. It is also determined how much sugar (either raw or refined) is transported between the storage points (factories of SCDS in Khuzestan, intermediate warehouses, external refineries, agents) and consumption points of the network (customers).

Note that in this model, the customers in different market segments are divided into two categories instead of the initial four market segments: (A) those who receive the needed sugar via from the warehouses of the agents and (B) those who receive their demand directly from the factories in Khuzestan (internal warehouses), External refineries or intermediate warehouses. The first category includes the whole demand of the customers who deal with the small agents and 25.2% of the demand of those who deal with the big agents. The second group contains the customers who receive their demand via syndicates or direct sale and also 74.8% of the demand of those who deal with the big agents.

Furthermore, agents themselves can receive the needed sugar directly from the factories in Khuzestan (internal warehouses), external refineries or intermediate warehouses. It is assumed that each small agent must assign a storage capacity of 1500 tons to the distribution network. For the big agents, this capacity is 1386 tons. The available storage capacity of the big agents is smaller than that of small agents, because most of the customers of the big agents (74.8%) receive sugar from other storage points of the network including intermediate warehouses, external refineries and internal warehouses in Khuzestan.

The decision variables of the second-level model are defined as follows:

- $Y_k = \begin{cases} 1 & \text{If a big (14000-ton capacity) intermediate warehouse is established in the } k \text{ th candidate location} \\ 0 & \text{otherwise} \end{cases}$
- $Y'_k = \begin{cases} 1 & \text{If a medium (10000-ton) intermediate warehouse is established in the } k \text{ th candidate location} \\ 0 & \text{otherwise} \end{cases}$
- $Y''_k = \begin{cases} 1 & \text{If a small (5000-ton) intermediate warehouse is established in the } k \text{ th candidate location} \\ 0 & \text{otherwise} \end{cases}$
- I_t: Inventory of refined sugar in the [internal] warehouses of the factories of SCDS in Khuzestan at the end of month t (tons) $t = 0...12$
- G_t: Inventory of raw sugar in the [internal] warehouses of SCDS in Khuzestan at the end of month t (tons) $t = 0...12$

The initial (at the beginning of the year where $t=0$) inventory of raw sugar in all of the internal warehouses is zero.

- P''_t : The amount of raw sugar which is refined in the [internal] refineries of SCDS in Khuzestan in month t (tons) $t=1...12$
- I'_{jt} : Inventory of refined sugar at the end of month t in the [external] refinery j (tons), $j=1...41$, $t=0...12$
- R_{jt} : Inventory of raw sugar at the end of month t in the [external] refinery j (tons), $j=1...41$, $t=0...12$

Only in the 6th external refinery, there is an inventory of 15000 tons of raw sugar at the beginning. $R_{6,0}=15000$

- L_{jt} : The amount of raw sugar which is refined in the [external] refinery j in month t (tons), $j=1...41$, $t=1...12$

Note that the amount of raw sugar is reduced by 7% when refined to white sugar.

- I''_{kt} : Inventory of refined sugar at the end of month t in the intermediate warehouse k (tons) (in case a warehouse is established in the candidate location k), $k=1...12$, $t=0...12$
- I'''_{it} : Inventory of refined sugar at the end of month t in the warehouses of the agents of the province i (tons), $i=1...30$, $t=0...12$

- $\boldsymbol{T}_{jt}$: the amount of raw sugar which is transported from factories of Khuzestan to the external refinery j in month t (tons), $j=1...40$, $t=1...12$

Knowing that it takes a month in the external refineries to have the raw sugar refined after receiving, $\boldsymbol{T}_{jt}$ also represents the amount of sugar which is refined in the external refinery j in month $t+1$.

- T'_{kt} : the amount of refined sugar which is transported from the factories of SCDS in Khuzestan to the intermediate warehouse k (in case a warehouse is established in the candidate location k) in month t (tons), $k=1...12$, $t=1...12$

- T''_{it} : the amount of refined sugar which is transported from the factories of SCDS in Khuzestan to the Group B customers in the province i in month t (tons), $i=1...30$, $t=1...12$
- T'''_{it} : the amount of refined sugar which is transported from the factories of SCDS in Khuzestan to the warehouses of the agents (Group A customers) of the province i (either big or small) in month t (tons), $i=1...30$, $t=1...12$
- Q_{jkt} : the amount of refined sugar which is transported from the external refinery j to the intermediate warehouse k (in case a warehouse is established in the candidate location k) in month t (tons), $j=1...40$, $k=1...12$, $t=1...12$
- Q'_{jit} : the amount of refined sugar which is transported from the external refinery j to the group B customers in the province i , in month t (tons), $j=1...40$, $i=1...30$, $t=1...12$
- Q''_{jit} : the amount of refined sugar which is transported from the external refinery j to the warehouses of the agents of the province i (either big or small) in month t (tons) (Group A customers), $j=1...40$, $i=1...30$, $t=1...12$
- X_{kit} : the amount of refined sugar which is transported from the intermediate warehouse k (in case a warehouse is established in the candidate location k) to the group B customers in month t (tons), $k=1...12$, $i=1...30$, $t=1...12$
- X'_{kit} : the amount of refined sugar which is transported from the intermediate warehouse k (in case a warehouse is established in the candidate location k) to the warehouses of the agents of the province i (either big or small) in the month t (tons) (group A customers), $k=1...12$, $i=1...30$, $t=1...12$

The available data of the second-level model are in the form of the following parameters:

1) Parameters related to capacity of storage and refining

- P'_t : the amount of raw sugar produced in the factories of SCDS in Khuzestan in month t (tons), $t=1...12$
- P_t : refining capacity of the [internal refineries] of SCDS in Khuzestan in month t (tons of raw sugar), $t=1...12$
- A' : total storage capacity of the raw sugar [internal] warehouses of SCDS (tons)
- A : total storage capacity of the refined sugar [internal] warehouses of SCDS (tons)

- C_{jt} : available refining capacity of the external refinery j in the month t (tons of raw sugar), $j=1...40$, $t=1...12$
- U_i : total storage capacity of the warehouses of all the big and small agents of the province i , $i=1...30$ (tons) $U_i = 1500x_i + 1386x_i'$
- W_k : storage capacity of a big intermediate warehouse, in case it is established in the candidate location k (tons), $k=1...12$
- W_k' : storage capacity of a medium intermediate warehouse, in case it is established in the candidate location k (tons), $k=1...12$
- W_k'' : storage capacity of a small intermediate warehouse, in case it is established in the candidate location k (tons), $k=1...12$
- $W_k = 14000$, $W_k' = 10000$, $W_k'' = 5000$ $\forall k$

2) Parameters related to demand:

- V_{it} : total demand of those market segments in the province i ,which do not receive it from the warehouses of the agents (group B customers), in month t (tons), $t=1...12$, $i=1...30$
- V_{it}' : total demand of those market segments in the province i ,which receive it from the warehouses of the agents (group A customers), in month t (tons), $t=1...12$, $i=1...30$

For the latter demand group (group A), whose demand is calculated based on the outcome of the first-level model, it is enough only to determine how sugar is transported to the agents. Agents themselves will decide how to deliver it to the final customers. However, for group B customers, the model must plan how to deliver the sugar to the final customers.

3) Cost parameters:

- H : holding cost of a ton of refined sugar per month in the [internal] warehouses of SCDS in Khuzestan
- $\overline{H}$: holding cost of a ton of raw sugar per month in the [internal] warehouses of SCDS in Khuzestan
- B : cost of refining a ton of raw sugar in the external refineries
- H_j' : holding cost of a ton of refined sugar per month in the warehouses of the [external] refinery j , $j=1...41$

- R_k : capital cost of establishing a big intermediate warehouse in the candidate location k per year, $k=1...12$
- R'_k : capital cost of establishing a medium intermediate warehouse in the candidate location k per year, $k=1...12$
- R''_k : capital cost of establishing a small intermediate warehouse in the candidate location k per year, $k=1...12$

The above capital costs do not depend on the amount of the stored sugar and the duration of storage. They can either be in the form of annual rent or calculated based on the interest rate and the investment cost.

- H''_k : holding cost of a ton of refined sugar per month in a big intermediate warehouse in the candidate location k , $k=1...12$
- $\tilde{H}_k$: holding cost of a ton of refined sugar per month in a medium intermediate warehouse in the candidate location k , $k=1...12$
- $\hat{H}_k$: holding cost of a ton of refined sugar per month in a small intermediate warehouse in the candidate location k , $k=1...12$
- H'''_i : holding cost of a ton of refined sugar per month in the warehouses of the agents of the province i , $i=1...30$
- D_j : transportation cost of a ton of raw sugar from the factories of SDCS in Khuzestan to the external refinery j , $j=1...40$
- D'_k : transportation cost of a ton of refined sugar from the factories of SDCS in Khuzestan to an intermediate warehouse in the candidate location k , $k=1...12$
- F_i : transportation cost of a ton of refined sugar from the factories of SDCS in Khuzestan to the to the center of the province i , $i=1...30$
- F''_{ji} : Transportation cost of a ton of refined sugar from the external refinery j to the center of the province i , $j=1...40$, $i=1...30$
- D''_{jk} : Transportation cost of a ton of refined sugar from the external refinery j to the intermediate warehouse k , $j=1...40$, $k=1...12$

- F'_{ki}: Transportation cost of a ton of refined sugar from the intermediate warehouse k to the center of the province i, $i=1...30$, $k=1...12$
- L: Cost of loading/unloading a ton of sugar from/to the trucks (anywhere)

The objective of the second-level model is to minimize the total annual cost including:

- The capital cost of establishing intermediate warehouses
- The holding (storage) cost of raw sugar in the warehouses of SCDS in Khuzestan and external refineries (note that according to the contracts at the time between SCDS and external refineries, there was no storage cost for raw sugar in the warehouses of the external refineries. The holding cost was included in the refining cost)
- The holding cost of refined sugar in the warehouses of SCDS in Khuzestan, external refineries, intermediate warehouses and agents (note that according to the contracts at the time between SCDS and external refineries and agents, there was no storage cost for refined sugar in the warehouses of the external refineries and agents. For the external refineries, the holding cost was included in the refining cost)
- The transportation cost of raw and refined sugar between:
    - Internal warehouses (Khuzestan) and external refineries
    - Internal warehouses and intermediate warehouses
    - Internal warehouses and agents
    - Internal warehouses and final customers of group B
    - External refineries and intermediate warehouses
    - External refineries and agents
    - External refineries and final customers of group B
    - Intermediate warehouses and agents
    - Intermediate warehouses and final customers of group B
- The loading and unloading costs of raw and refined sugar in the external refineries and intermediate warehouses (Loading and unloading costs are fixed in the other storage points of the network, because the production and demand rates are known and fixed.)

The following constraints must be formulated in the second-level model:

Inventories of raw and refined sugar at [the end of] each month at each storage point of the distribution network (internal warehouses, external refineries, intermediate warehouses and

agents) cannot exceed the storage capacity. Also, the refining capacity in the internal and external refineries is limited. Both intermediate warehouses and warehouses of the agents can only store refined sugar. Therefore, the following constraints must be considered:

- Capacity of the warehouses of raw sugar in Khuzestan
- Capacity of the [internal] warehouses of refined sugar in Khuzestan (warehouses of the factories of SCDS)
- Capacity of the warehouses of the agents (refined sugar)
- Refining capacity of the [internal] factories of SCDS in Khuzestan and the relation between the inventory of raw sugar and the amount of raw sugar refined in the internal refineries
- Raw sugar cannot be kept for more than 2 months in the warehouses of the company in Khuzestan.
- Refining capacity of the external refineries
- Any shipment of raw sugar to external refineries must be refined within at most two months. i.e. Raw sugar cannot be kept in the warehouse of an external refinery for more than two months.
- Storage limit of the intermediate warehouses
- Only one type of intermediate warehouse can be established in each candidate location.
- Sugar can be transported to an intermediate warehouse only if actually an intermediate warehouse is established in the corresponding candidate location.
- Maximum storage time of refined sugar in the warehouses of the external refineries is one month. Consider the 7% reduction of raw sugar in weight when refined. Furthermore, it is assumed that external refineries have no limit on storage capacity of refined sugar.
- Demand satisfaction constraint for the customers who receive their sugar from the warehouses of the agents

  Note that a customer may receive service from the agents, but receives the needed sugar directly from an intermediate warehouse or refinery.

- Demand satisfaction constraint for the customers who do not receive their sugar from the warehouses of the agents (group B)
- Inventory balance constraint of raw sugar in the warehouses of the factories of the company (SCDS) at the end of each month
- Inventory balance constraint of refined sugar in the warehouses of the factories of the company (SCDS) at the end of each month

- Inventory balance constraint of refined sugar in the warehouses of the external refineries at the end of each month
- Inventory balance constraint of raw sugar in the warehouses of the external refineries at the end of each month
- Inventory balance constraint of the intermediate warehouses at the end of each month
- Inventory balance constraint of the warehouses of the agents at the end of each month

  In all of the inventory balance constraints at each storage point, the relation between inventory at the end of each month with the inventory at the end of the previous month, transportations to/from the storage point and consumption/refining must be formulated.

- Initial inventories

All of the relations including the objective function and constraints can be formulated in linear form.