Maintenance scheduling optimization

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Maintenance optimization — a background

- Invitation 2000 from Volvo Aero Corporation (VAC, nowadays GKN Aerospace): maintenance of the RM12 jet engine
- Paired PhD project between applied math/optimization and math statistics/material fatigue and reliability
- Optimization student: a model for opportunistic maintenance; superior to simpler policies
- Math statistics student: models for the determination of life distributions based on crack growth
- Continuation projects: GKN; planning maintenance of components in wind power plants and scheduling of rail grinding

A conversation with Bo Hägg, CEO Underhållsföretagen

- Maintenance = obtain reliability at the least cost
- Maint. costs/year: 14K Billion SEK (EU), 275 Billion SEK (S)
- Maintenance is often seen merely as a cost
- Maintenance is sometimes done too often—inspections and measurements may damage the systems
- Sometimes—like with road/rail infrastructure and "Miljonprogramhusen'—it is performed seldom
- Truth: well performed maintenance is an investment in availability and safety

Maintenance principles

- Preventive maintenance: actions that prevent failure
- Corrective maintenance: actions after failure, repairs
- Condition based maintenance: measurements $\rightarrow$ predictions $\rightarrow$ actions according to a maintenance principle
- Opportunistic maintenance: when maintenance must be performed, make also some (additional) preventive maintenance actions

A simple example, I

- A system with n components
- Life of component i : T_i time units (intervals)
- Time horizon: T time units (e.g. contract period)
- Cost of a spare component of type i at time t: c_{it} monetary units
- Cost for performing any maintenance at time t: d_t monetary units

A simple example, II

- Variables are logical – do something or not
- Model uses binary variables:

$$x_t = \begin{cases} 1, & \text{if "something" is done at time } t \\ 0, & \text{otherwise} \end{cases}$$

- A decision often implies other necessary decisions
- Example: if component i shall be replaced at time t maintenance must be performed
- Such logical relations are equivalent to linear constraints:

$$\text{if A then B} \quad \Longleftrightarrow \quad x_A \leq x_B$$

The basic replacement problem, I

- Goal: minimize the total cost for a working system during the contract period:

Mathematical model

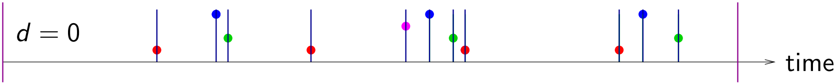
$$
\begin{aligned}
&\underset{(x,z)}{\text{minimize}} && \sum_{t=1}^{T} \left(\sum_{i=1}^{N} c_{it} x_{it} + d_t z_t \right), && && \text{(1a)}\\
&\text{subject to} && \sum_{t=\ell+1}^{\ell+T_i} x_{it} \geq 1, && \ell = 0, \dots, T - T_i, \; i = 1, \dots, N, && \text{(1b)}\\
& && x_{it} \leq z_t, && t = 1, \dots, T, \; i = 1, \dots, N, && \text{(1c)}\\
& && x_{it} \geq 0, && t = 1, \dots, T, \; i = 1, \dots, N, && \text{(1d)}\\
& && z_t \leq 1, && t = 1, \dots, T, && \text{(1e)}\\
& && x_{it}, z_t \in \{0, 1\}, && t = 1, \dots, T, \; i = 1, \dots, N && \text{(1f)}
\end{aligned}
$$

The basic replacement problem, II

- Objective (1a): minimize the total cost of having a working system during the contract period
- Constraint (1b): for any given item i in the system, the component must be replaced at some point during *every* time interval of T_i time steps
- Constraint (1c): we cannot perform any replacement without paying the fixed cost d_t for performing a maintenance operation; once we do pay, any maintenance action becomes possible (at no extra fixed cost) at that time step
- Constraints (1d)–(1f) ensure that the variables take only meaningful values

Opportunistic maintenance or not?

- Example: four components with different prices and lives
- A replacement is marked with a dot; its colour represents the type of component replaced



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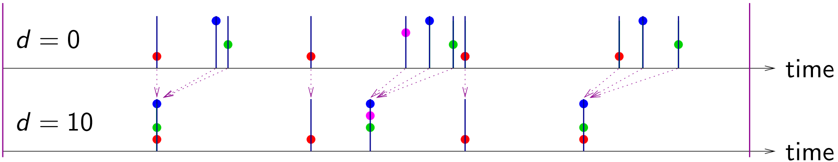
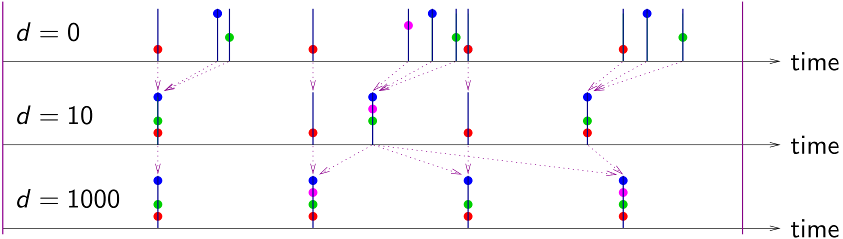


Opportunistic maintenance or not?

- Example: four components with different prices and lives
- A replacement is marked with a dot; its colour represents the type of component replaced
- The larger the fixed cost, the more beneficial sl opportunistic maintenance becomes; also more items are replaced



Constraint structure—example

- Component 3: $\displaystyle\sum_{t=\ell+1}^{\ell+T_3} x_{3t} \geq 1, \quad \ell = 0, \dots, T - T_3$
- $T = 8,\ T_3 = 4 \quad \Longrightarrow \quad \displaystyle\sum_{t=\ell+1}^{\ell+4} x_{3t} \geq 1, \quad \ell = 0, \dots, 4$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_{31} \\ x_{32} \\ \vdots \\ x_{38} \end{bmatrix} \geq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Property I: the replacement problem is NP-hard

Theorem

Set covering is polynomially reducible to the replacement problem

- This essentially mean that we *cannot* expect to find an optimal solution in a time that is proportional to a polynomial function of the problem size ($T(N+1)$ variables and $\approx 4NT$ constraints)
- Basic complexity theory: Chapter 2.6 in the course book

Property II: with fixed z the problem over x is easy

- The constraint matrix has the "consecutive ones" property

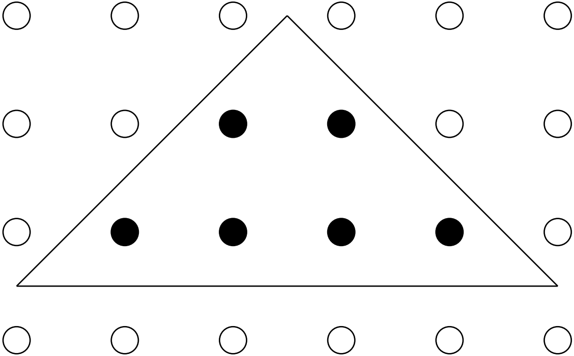
$\Rightarrow$ For fixed values of z, the problem over x can be solved as a linear program

- For each i, the linear programming dual problem can be solved by a "greedy" algorithm $\Rightarrow$ primal solution by complementarity; see [a], Algorithm 1, page 297
- The latter is typically 5–40 times faster than solving as a general linear program, and 25–400 times faster when costs are monotone with time (i.e., $\forall t$ either $c_{it} \leq c_{i,t+1}$ or $c_{it} \geq c_{i,t+1}$); see [a], Algorithm 2, page 299

[a] T. Almgren, N. Andréasson, M. Patriksson, A.-B. Strömberg, A. Wojciechowski, M. Önnheim (2012): *The opportunistic replacement problem: theoretical analyses and numerical tests*, Mathematical Methods of Operations Research, 76(3) pp. 289–319. http://link.springer.com/article/10.1007%2Fs00186-012-0400-y

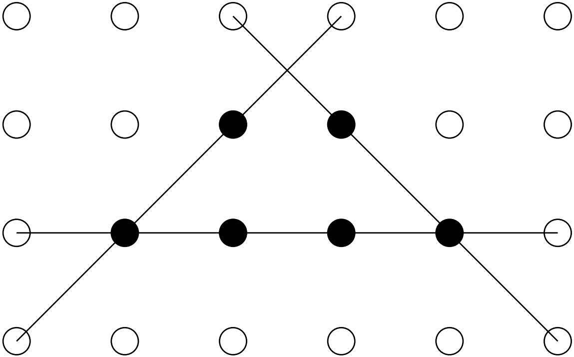
Property III: all inequalities are facet defining

No inequalities are facet defining



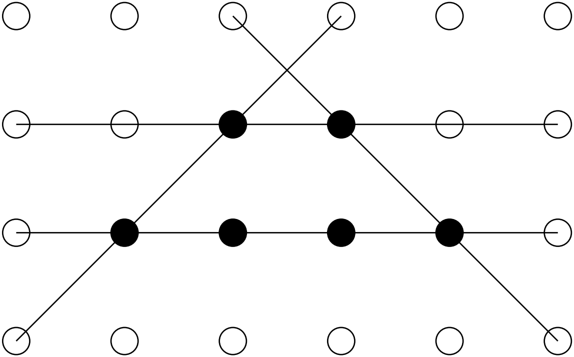
Property III: all inequalities are facet defining

All inequalities are facet defining



Property III: all inequalities are facet defining

Integral polyhederon



See [a], Section 5.1–5.2

A generalized model

New variable definition

Define the set

$$\mathcal{I} := \{\, (s,t) \mid 0 \le s < t \le T+1;\ s,t \in Z \,\}$$

of replacement intervals and introduce the variables

$$x^i_{st} = \begin{cases} 1, & \text{if component } i \text{ receives PM at the times } s \text{ and } t\text{, and not in-between,} \\ 0, & \text{otherwise,} \end{cases} \qquad i \in \mathcal{N},\ (s,t) \in \mathcal{I},$$

and

$$z_t = \begin{cases} 1, & \text{if maintenance occurs at time } t, \\ 0, & \text{otherwise,} \end{cases} \qquad t \in \mathcal{T}.$$

A generalized model

$$
\begin{aligned}
\text{minimize} \quad & \sum_{t\in\mathcal{T}} d_t z_t + \sum_{i\in\mathcal{N}} \sum_{(s,t)\in\mathcal{I}} c^i_{st} x^i_{st}, && && \text{(2a)}\\
\text{subject to} \quad & \sum_{s=0}^{t-1} x^i_{st} \le z_t, && i \in \mathcal{N},\ t \in \mathcal{T}, && \text{(2b)}\\
& \sum_{s=0}^{t-1} x^i_{st} = \sum_{r=t+1}^{T+1} x^i_{tr}, && i \in \mathcal{N},\ t \in \mathcal{T}, && \text{(2c)}\\
& \sum_{t=1}^{T+1} x^i_{0t} = 1, && i \in \mathcal{N}, && \text{(2d)}\\
& x^i_{st} \in \{0,1\}, && i \in \mathcal{N},\ (s,t) \in \mathcal{I}, && \text{(2e)}\\
& z_t \in \{0,1\}, && t \in \mathcal{T}. && \text{(2f)}
\end{aligned}
$$

On the GKN project

- Aircraft engines are expensive:
  - Spare components cost up to 2 MSEK
  - Total cost of maintenance of one engine: 15–30 MSEK
  - Maximizing “time on wing” is important, both for civil and military aircraft
- The aircraft engine RM12 consists of 7 modules and 61 components in total
- A mathematical model has been constructed for the entire engine maintenance, including work costs for (dis)assembling the necessary modules and components for each maintenance occasion
- This model has slightly less than 6000 binary variables

Results on the GKN problems

- An individual engine module with 10 components: cost reduction 35%; reduction of # maint. occasions 7% (compared with a simple policy similar to that used at GKN)
- Complete engine of 7 modules (61 components):
  - Cost reduction compared to maintaining (optimally) each individual module: 12%
  - Reduction of # maint. occasions: 60%
- Product development: found 5 components that can potentially reduce maintenance costs more than 5% through prolonged lives

Maintenance of rails and wheels, I

- Paired PhD project in collaboration with CHARMEC
- Background: increased wear of rails and wheels due to an increase in speeds and loads
- Aim to develop decision support tools for the optimization of inspection and maintenance of rails and wheels wrt. LCC, safety and maintenance logistics
- The other PhD student (at CHARMEC) models the progressive degradation of rails and wheels
- Will result in advanced knowledge on how component condition indicators can be efficiently used in an optimization

Maintenance of rails and wheels, II

- The picture below shows how a component of a rail is degraded over time (measured in portions of the rail sections having crack lengths in given intervals)
- Several levels of maintenance can be performed, with different effects (and different costs)
- The optimization will determine which maintenance action is the most appropriate at any given time