MVE165/MMG631
Linear and Integer Optimization with Applications
Lecture 2
AMPL and CPLEX, Assignment 1

Zuzana Šabartová

2014–03–21

Overview

1. AMPL
2. CPLEX
3. Simple problem
4. Assignment 1
5. References

AMPL

- Algebraic modeling language for linear and nonlinear optimization problems
- Formulate optimization models and examine solutions
- Manage communication with an appropriate solver
- Natural syntax
- Separation of model and data
- Discrete or continuous variables
- Support for sets and set operators
- Built in arithmetic functions
- Looping, if-then-else commands

Solvers that work with AMPL

- **CPLEX** - linear and quadratic problems in continuous and integer variables
- Gurobi - linear and quadratic problems in continuous and integer variables
- CONOPT - nonlinear problems
- MINOS - linear and nonlinear problems
- CONDOR, Gecode, IPOPT, MINLP, SNOPT ...

Overview

CPLEX

- Optimization software package for solving linear and quadratic problems in continuous and integer variables
- Originally based on simplex method implemented in C
- Primal and dual simplex method
- Barrier method
- Techniques to avoid degeneracy
- Cutting planes
- Branch & Bound algorithm
- Heuristics ...

Overview

1. AMPL
2. CPLEX
3. Simple problem
4. Assignment 1
5. References

The Diet Problem - description

- G. B. Dantzig, 1990
- Choose prepared foods to meet certain nutritional requirements in the cheapest way
- Precooked foods (beef, chicken, fish) are available in given quantity and for given price
- Each food provide given percentage of daily requirements of nutrients (A, C, B1, B2)

	Price [$]	Av. [pcs]	A [%]	C [%]	B1 [%]	B2 [%]
Beef	3.19	22	60	20	10	15
Chicken	2.59	48	8	0	20	20
Fish	2.29	45	8	10	15	10

- Demand: meet week's requirements, 700 % of daily requirements for each nutrient

The Diet Problem - model

The Diet Problem - model

- Sets:
  - $I = 1, 2, 3$ - set of kinds of food
  - $J = 1, 2, 3, 4$ - set of nutrients

The Diet Problem - model

- Sets:
  - $I = 1, 2, 3$ - set of kinds of food
  - $J = 1, 2, 3, 4$ - set of nutrients
- Variables:
  - $x_i, \ i = 1, \dots, 3$ - purchased amount of food i [pcs]

The Diet Problem - model

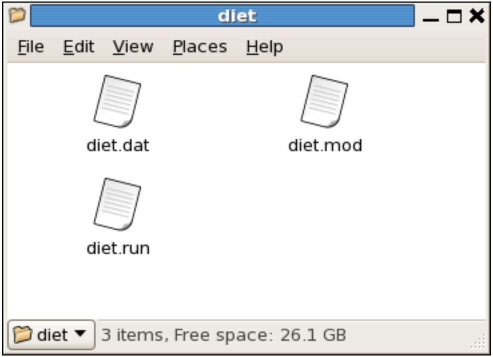
- Sets:
  - $I = 1, 2, 3$ - set of kinds of food
  - $J = 1, 2, 3, 4$ - set of nutrients
- Variables:
  - $x_i, \; i = 1, \ldots, 3$ - purchased amount of food i [pcs]
- Parameters:
  - $c_i, \; i = 1, \ldots, 3$ - cost of one piece of food i [$]
  - $a_i, \; i = 1, \ldots, 3$ - available amount of food i [pcs]
  - $p_{ij}, \; i = 1, \ldots, 3, \; j = 1, \ldots, 4$ - percentage of daily requirement of nutrient j in food i [%]
  - d - requirement for nutrients [%]

The Diet Problem - model

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{3} c_i x_i \\
\text{s.t.} \quad & \sum_{i=1}^{3} p_{ij}\, x_i \geq d_j, \quad && j = 1, \ldots, 4 \\
& x_i \leq a_i, \quad && i = 1, \ldots, 3 \\
& x_i \geq 0, \quad && i = 1, \ldots, 3
\end{aligned}
$$

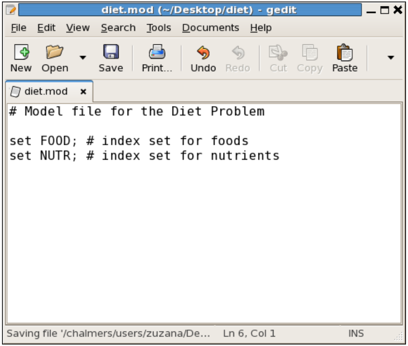
The Diet Problem - AMPL implementation

- Create folder *diet*
- Create model file *diet.mod*
- Create data file *diet.dat*
- Create run file *diet.run*



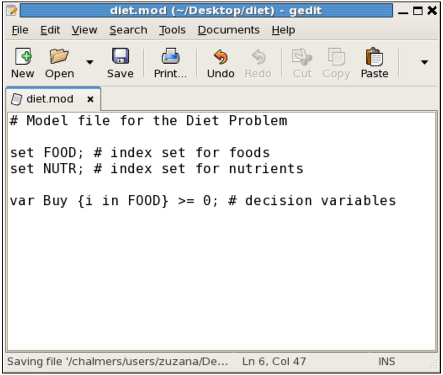
The Diet Problem - AMPL implementation

- Fill the model file using text editor (Emacs, gedit, ...)
- Introduce index sets: *set*
- Comments start with #, each command ends with ;
- Sets:
  - $I = 1, 2, 3$
  - $J = 1, 2, 3, 4$



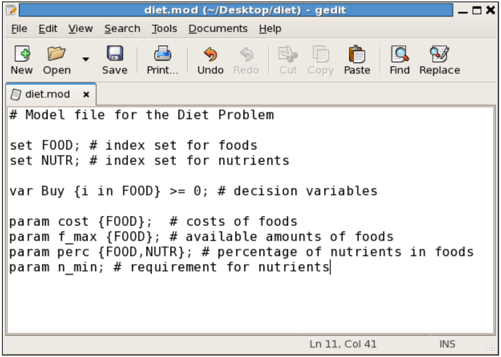
The Diet Problem - AMPL implementation

- Introduce variables: *var*
- Formulate non-negativity requirements
- Variables:
  - $x_i, \;\; i = 1, \ldots, 3$
  - $x_i \geq 0, \;\; i = 1, \ldots, 3$



The Diet Problem - AMPL implementation

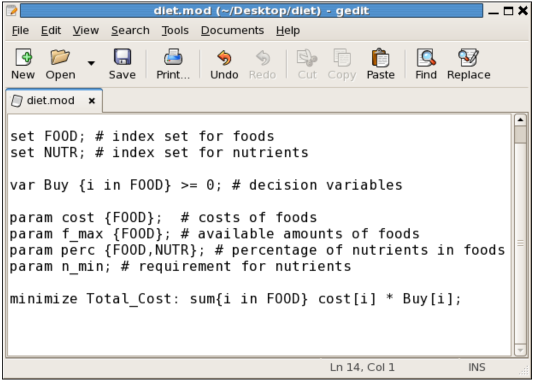
- Introduce parameters: *param*
- Parameters:
  - $c_i, \ i = 1, \dots, 3$
  - $a_i, \ i = 1, \dots, 3$
  - $p_{ij}, \ i = 1, \dots, 3, \ j = 1, \dots, 4$
  - d




```
# Model file for the Diet Problem

set FOOD; # index set for foods
set NUTR; # index set for nutrients

var Buy {i in FOOD} >= 0; # decision variables

param cost {FOOD};  # costs of foods
param f_max {FOOD}; # available amounts of foods
param perc {FOOD,NUTR}; # percentage of nutrients in foods
param n_min; # requirement for nutrients
```

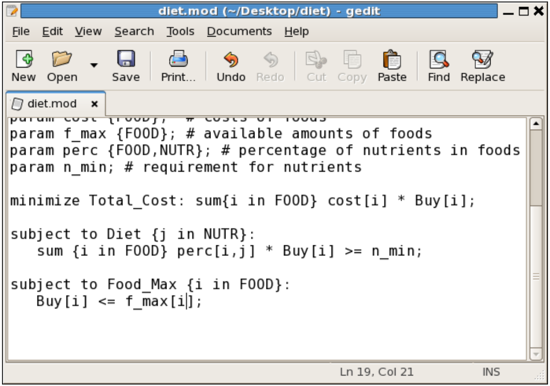
The Diet Problem - AMPL implementation

- Formulate objective function: *minimize, maximize*
- Use built-in arithmetic functions: $+$, $-$, $*$, $^\wedge$, $/$, *sum, prod, abs, log, sin ...*
- min $\sum_{i=1}^{3} c_i x_i$



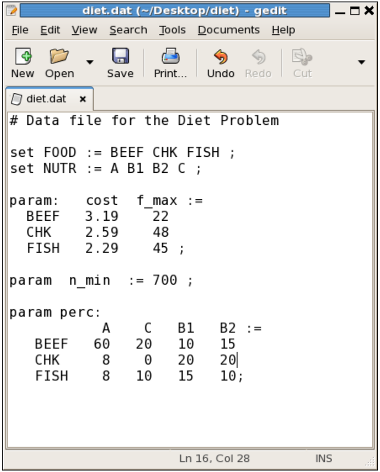
The Diet Problem - AMPL implementation

- Formulate constraints: *subject to*
- Use arithmetic relations: $>$, $>=$, $<$, $<=$, $==$, $!=$ ...
- $\sum_{i=1}^{3} p_{ij} x_i \geq d_j, \;\; j = 1, \ldots, 4$
- $x_i \leq a_i, \;\; i = 1, \ldots, 3$



The Diet Problem - AMPL implementation

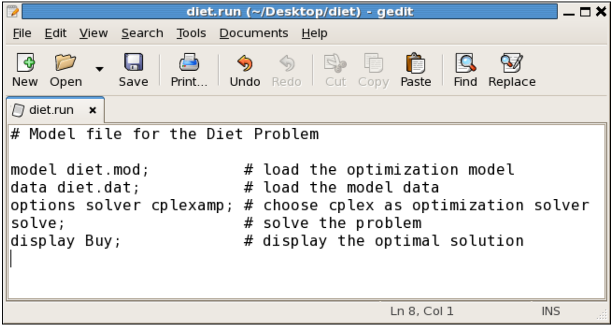
- Fill the data file using text editor
- Assign values to introduced sets and parameters




```
# Data file for the Diet Problem

set FOOD := BEEF CHK FISH ;
set NUTR := A B1 B2 C ;

param:   cost   f_max :=
  BEEF   3.19    22
  CHK    2.59    48
  FISH   2.29    45 ;

param  n_min  := 700 ;

param perc:
           A    C    B1    B2 :=
   BEEF   60   20    10    15
   CHK     8    0    20    20
   FISH    8   10    15    10;
```

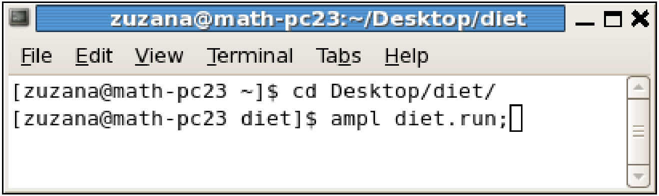
The Diet Problem - AMPL implementation

- Fill the run file using text editor
- Load the model and data: *model*, *data*
- Choose solver: *options solver*
- Solve the problem: *solve*
- Display results: *display*



The Diet Problem - AMPL implementation

- Open Terminal
- Go to the folder *diet*
- Evaluate commands in run file *diet.run* by AMPL



The Diet Problem - AMPL implementation

zuzana@math-pc23:~/Desktop/diet

File Edit View Terminal Tabs Help

```
[zuzana@math-pc23 ~]$ cd Desktop/diet/
[zuzana@math-pc23 diet]$ ampl diet.run;
IBM ILOG License Manager: "university-goteborg" is accessing AMPL 12.
IBM ILOG AMPL 12.1.0 (5724-Y45) AMPL Version 20090327 (Linux 2.6.18-6-amd64)
IBM ILOG License Manager: "university-goteborg" is accessing CPLEX 12 with optio
n(s): "b e m q use=10 ".
CPLEX 12.1.0: optimal solution; objective 143.965
4 dual simplex iterations (0 in phase I)
Buy [*] :=
BEEF   22
 CHK    5.5
FISH   26
;

[zuzana@math-pc23 diet]$ 
```

The Diet Problem - AMPL implementation

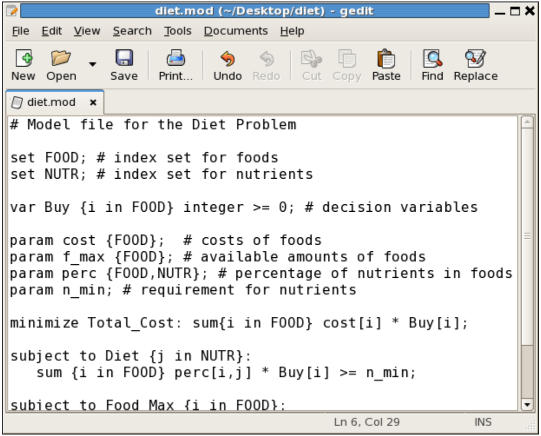
- Perform sensitivity analysis
- Preserve the sensitivity analysis information
- Use suffices for sensitivity analysis: *.rc, .slack, .dual, ...*

diet.run (~/Desktop/diet) - gedit

```
# Model file for the Diet Problem

model diet.mod;            # load the optimization model
data diet.dat;             # load the model data
options solver cplexamp; # choose cplex as optimization solver

option cplex_options 'sensitivity'; # preserve the sensitivity
option presolve 0;                  # analysis information
option solve;                       # do not reduce the problem

solve;                     # solve the problem
display Buy;               # display the optimal solution
display Diet.dual;         # display dual variables
display Food_Max.slack;    # display slack variables
display Buy.rc;            # display reduced costs
```

The Diet Problem - AMPL implementation

- Change type of variables: *integer, binary, ...*
- Sensitivity analysis as described is possible for linear programs in continuous variables



The Diet Problem - AMPL implementation

zuzana@math-pc23:~/Desktop/diet

File Edit View Terminal Tabs Help

```
[zuzana@math-pc23 ~]$ cd Desktop/diet/
[zuzana@math-pc23 diet]$ ampl diet.run;
IBM ILOG License Manager: "university-goteborg" is accessing AMPL 12.
IBM ILOG AMPL 12.1.0 (5724-Y45) AMPL Version 20090327 (Linux 2.6.18-6-amd64)
IBM ILOG License Manager: "university-goteborg" is accessing CPLEX 12 with optio
n(s): "b e m q use=10 ".
CPLEX 12.1.0: optimal integer solution; objective 144.96
2 MIP simplex iterations
0 branch-and-bound nodes
1 mixed-integer rounding cut
Buy [*] :=
BEEF  22
 CHK   5
FISH  27
;

[zuzana@math-pc23 diet]$
```

Student representatives

Randomly selected course representatives that will evaluate the course:

- Jessica Fredby
- Jonas Jagers
- Johan Karlsson
- Joacim Linder
- Alexander Lyckell

Overview

Biofuels supply chain

- Reduce oil dependence
- Reduce greenhouse effect and climate change
- Substitute fuel in transportation sector
- Biofuels can be used in existing cars
- EU quotas to use 10 % of energy from renewable sources in transportation by 2020, to use 5 % of biodiesel in diesel fuel from 2003
- Food versus fuel debate
- Need to develop a mathematical model of the biofuels supply chain

Biofuels supply chain

The value chain typically includes:

- Feedstock production
- Biofuel production
- Blending
- Distribution
- Consumption

Assignment 1: Biodiesel supply chain

- Biodiesel supply chain problem
- Maximize the total profit
- Supply the demand of biodiesel
- Tasks:
  - Formulate linear optimization model
  - Model and solve the problem using AMPL and CPLEX
  - Perform sensitivity analysis

Crops

- Data:
  - Available area
  - Soya, Sunflower, Cotton
  - Each crop yields expected amount of seeds
  - Each crop has water demand
  - Available water
- Processes
  - Extraction of vegetable oils from seeds (given yields)
  - Transesterification: vegetable oil + methanol = biodiesel (given proportions)
  - Purchase methanol (given price)

Final Products

- Data:
  - B5, B30, B100
  - Each product has price
  - Each product has tax (higher amount of biodiesel $\Rightarrow$ lower tax)
  - Demand of fuels to be delivered
- Processes
  - Blending of biodiesel and petrol diesel
  - Purchase petrol diesel (given price and availability)

Sensitivity analysis

- Analyze results and answer several important questions without changing the model.
- How sensitive is the optimal solution to changes in data values?
  - *Reduced costs* of a non-basic variables: the change in the objective value when the value of the corresponding variable is (marginally) increased
  - *Shadow price* of a constraint: the change in the optimal value if you change (marginally) the RHS, it equals to the optimal value of the corresponding dual variable
  - *Slack variables* of a constraint: indicate how much the RHS can be reduced while staying feasible
- Use these tools to answer the questions

Cetane number

- Quality of pure biodiesel is given by cetane number
- Quality of petrol diesel is given by octane number
- Octane-cetane relationship is linear
- Requirement for quality of each product should be incorporated

Overview

1. AMPL
2. CPLEX
3. Simple problem
4. Assignment 1
5. References

- R. Fourer, D.M. Gay, and B.W. Kernighan, *AMPL: A Modeling Language for Mathematical Programming*, Duxbury Press, 2003, `http://www.ampl.com/BOOK/download.html`.
- *IBM ILOG AMPL, Version 12.2, User's Guide, Standard (Command-line) Version Including CPLEX Directives*, IBM, May 2010, `http://www.ampl.com/BOOKLETS/amplcplex122userguide.pdf`.
- Z. Šabartová, A. B. Strömberg, *Assignment 1: Biodiesel supply chain*, March 7, 2014, `http://www.math.chalmers.se/Math/Grundutb/CTH/mve165/1314/`.
- Ch. Papapostolou, E. Kondili, J. K. Kaldellis, *Development and implementation of an optimisation model for biofuels supply chain*, Energy, Volume 36, Issue 10, October 2011, Pages 6019–6026.
- J. Lundgren, M. Rönnqvist, P. Värbrand, *Optimization*, Studentlitteratur AB, Lund, 2010.