

MVE165/MMG631
Linear and Integer Optimization with Applications
Lecture 2
AMPL and CPLEX, Assignment 1

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Overview

- 1 AMPL
- 2 CPLEX
- 3 Simple problem
- 4 Assignment 1
- 5 References

AMPL

- Algebraic modelling language for optimization problems
 - ⇒ Interface between problems and solvers
 - ⇒ Formulate optimization models and examine solutions
 - ⇒ Manage communication with an appropriate solver
- Natural syntax
- Separation of model and data
- Support for sets and set operators
- Built in arithmetic functions
- Looping, if-then-else commands

Solvers that work with AMPL

- **CPLEX** - linear and quadratic problems in continuous and integer variables
- Gurobi - linear and quadratic problems in continuous and integer variables
- CONOPT - nonlinear problems in continuous variables
- MINOS - linear and nonlinear problems in continuous variables
- CONDOR, Gecode, IPOPT, MINLP, SNOPT, etc.

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CPLEX

- Optimization software package for solving linear and quadratic problems in continuous and integer variables
- Originally based on simplex method implemented in C
- Primal and dual simplex method
- Barrier method
- Techniques to avoid degeneracy
- Cutting planes
- Branch & Bound algorithm
- Heuristics

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The Diet Problem - description

- G. B. Dantzig, 1990
- Choose prepared foods to meet certain nutritional requirements in the cheapest way
- Precooked foods (beef, chicken, fish) are available in given quantity and for given price
- Each food provide given percentage of daily requirements of nutrients (A, C, B1, B2)

	Price [\$]	Av. [pcs]	A [%]	C [%]	B1 [%]	B2 [%]
Beef	3.19	22	60	20	10	15
Chicken	2.59	48	8	0	20	20
Fish	2.29	45	8	10	15	10

- Demand: meet week's requirements, 700 % of daily requirements for each nutrient

The Diet Problem - mathematical model

The Diet Problem - mathematical model

- Sets:
 - $I = 1, 2, 3$ - set of kinds of food
 - $J = 1, 2, 3, 4$ - set of nutrients

The Diet Problem - mathematical model

- Sets:
 - $I = 1, 2, 3$ - set of kinds of food
 - $J = 1, 2, 3, 4$ - set of nutrients
- Variables:
 - $x_i, i = 1, \dots, 3$ - purchased amount of food i [pcs]

The Diet Problem - mathematical model

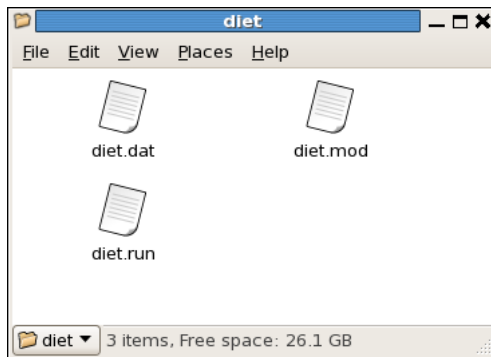
- Sets:
 - $I = 1, 2, 3$ - set of kinds of food
 - $J = 1, 2, 3, 4$ - set of nutrients
- Variables:
 - $x_i, i = 1, \dots, 3$ - purchased amount of food i [pcs]
- Parameters:
 - $c_i, i = 1, \dots, 3$ - cost of one piece of food i [\$]
 - $a_i, i = 1, \dots, 3$ - available amount of food i [pcs]
 - $p_{ij}, i = 1, \dots, 3, j = 1, \dots, 4$ - percentage of daily requirement of nutrient j in food i [%]
 - d - requirement for nutrients [%]

The Diet Problem - model

$$\begin{aligned} \min \quad & \sum_{i=1}^3 c_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^3 p_{ij} x_i \geq d, \quad j = 1, \dots, 4 \\ & x_i \leq a_i, \quad i = 1, \dots, 3 \\ & x_i \geq 0, \quad i = 1, \dots, 3 \end{aligned}$$

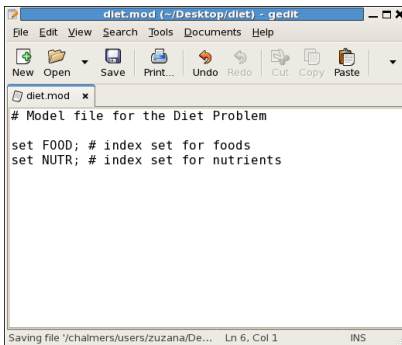
The Diet Problem - AMPL implementation

- Create folder *diet*
- Create model file *diet.mod*
- Create data file *diet.dat*
- Create run file *diet.run*



The Diet Problem - AMPL implementation

- Fill the model file using text editor (Emacs, gedit, ...)
- Introduce index sets: *set*
- Comments start with *#*, each command ends with *;*
- Sets:
 - $I = 1, 2, 3$
 - $J = 1, 2, 3, 4$



The screenshot shows a gedit window titled "diet.mod (~/Desktop/diet) - gedit". The window contains the following text:

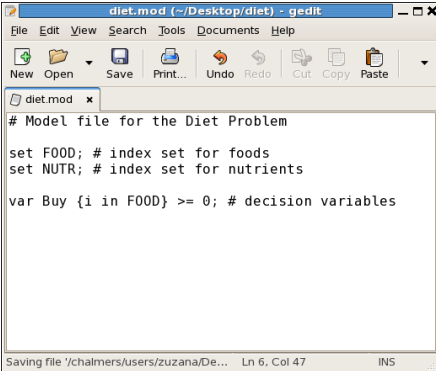
```
# Model file for the Diet Problem

set FOOD; # index set for foods
set NUTR; # index set for nutrients
```

The status bar at the bottom of the window indicates "Saving file '/chalmers/users/zuzana/De... Ln 6, Col 1 INS".

The Diet Problem - AMPL implementation

- Introduce variables: *var*
- Formulate non-negativity requirements
- Variables:
 - $x_i, i = 1, \dots, 3$
 - $x_i \geq 0, i = 1, \dots, 3$



```
# Model file for the Diet Problem

set FOOD; # index set for foods
set NUTR; # index set for nutrients

var Buy {i in FOOD} >= 0; # decision variables
```

The screenshot shows a gedit window titled "diet.mod (~/Desktop/diet) - gedit". The window contains the following AMPL code:

```
# Model file for the Diet Problem

set FOOD; # index set for foods
set NUTR; # index set for nutrients

var Buy {i in FOOD} >= 0; # decision variables
```

The status bar at the bottom of the window indicates "Saving file '/chalmers/users/zuzana/De... Ln 6, Col 47 INS".

The Diet Problem - AMPL implementation

- Introduce parameters: *param*
- Parameters:
 - $c_i, i = 1, \dots, 3$
 - $a_j, i = 1, \dots, 3$
 - $p_{ij}, i = 1, \dots, 3, j = 1, \dots, 4$
 - d

The screenshot shows a gedit window titled 'diet.mod (~/Desktop/diet) - gedit'. The window contains the following AMPL code:

```
# Model file for the Diet Problem

set FOOD; # index set for foods
set NUTR; # index set for nutrients

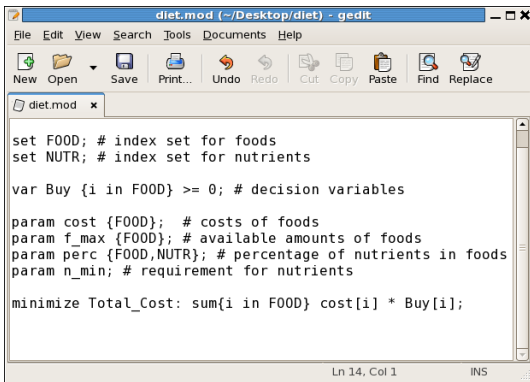
var Buy {i in FOOD} >= 0; # decision variables

param cost {FOOD}; # costs of foods
param f_max {FOOD}; # available amounts of foods
param perc {FOOD,NUTR}; # percentage of nutrients in foods
param n_min; # requirement for nutrients
```

The status bar at the bottom of the window indicates 'Ln 11, Col 41' and 'INS'.

The Diet Problem - AMPL implementation

- Formulate objective function: *minimize, maximize*
- Use built-in arithmetic functions: $+$, $-$, $*$, $^$, $/$, *sum*,
prod, *abs*, *log*, *sin*, ...
- $\min \sum_{i=1}^3 c_i x_i$



```

diet.mod (~/.Desktop/diet) - gedit
File Edit View Search Tools Documents Help
New Open Save Print... Undo Redo Cut Copy Paste Find Replace
diet.mod x
set FOOD; # index set for foods
set NUTR; # index set for nutrients

var Buy {i in FOOD} >= 0; # decision variables

param cost {FOOD}; # costs of foods
param f_max {FOOD}; # available amounts of foods
param perc {FOOD,NUTR}; # percentage of nutrients in foods
param n_min; # requirement for nutrients

minimize Total_Cost: sum{i in FOOD} cost[i] * Buy[i];
Ln 14, Col 1 INS

```

The Diet Problem - AMPL implementation

- Formulate constraints: *subject to*
- Use arithmetic relations: $>$, $>=$, $<$, $<=$, $==$, $!=$, ...
- $\sum_{i=1}^3 p_{ij}x_i \geq d$, $j = 1, \dots, 4$
- $x_i \leq a_i$, $i = 1, \dots, 3$

```

diet.mod (~/Desktop/diet) - gedit
File Edit View Search Tools Documents Help
New Open Save Print... Undo Redo Cut Copy Paste Find Replace
diet.mod x
param cost {FOOD}; # costs of foods
param f_max {FOOD}; # available amounts of foods
param perc {FOOD,NUTR}; # percentage of nutrients in foods
param n_min; # requirement for nutrients

minimize Total_Cost: sum{i in FOOD} cost[i] * Buy[i];

subject to Diet {j in NUTR}:
    sum {i in FOOD} perc[i,j] * Buy[i] >= n_min;

subject to Food_Max {i in FOOD}:
    Buy[i] <= f_max[i];

Ln 19, Col 21  INS

```

The Diet Problem - AMPL implementation

- Fill the data file using text editor
- Assign values to introduced sets and parameters

The screenshot shows a gedit window titled 'diet.dat (~/Desktop/diet) - gedit'. The window contains the following AMPL data file content:

```
# Data file for the Diet Problem

set FOOD := BEEF CHK FISH ;
set NUTR := A B1 B2 C ;

param: cost f_max :=
  BEEF 3.19 22
  CHK 2.59 48
  FISH 2.29 45 ;

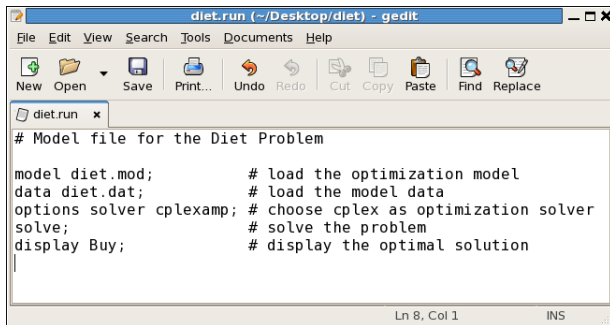
param n_min := 700 ;

param perc:
  A C B1 B2 :=
  BEEF 60 20 10 15
  CHK 8 0 20 20
  FISH 8 10 15 10;
```

The status bar at the bottom of the window indicates 'Ln 16, Col 28' and 'INS'.

The Diet Problem - AMPL implementation

- Fill the run file using text editor
- Load the model and data: *model, data*
- Choose solver: *options solver*
- Solve the problem: *solve*
- Display results: *display*



The screenshot shows a gedit window titled "diet.run (~/Desktop/diet) - gedit". The window contains the following AMPL run file code:

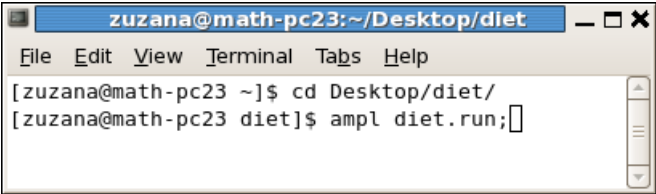
```
# Model file for the Diet Problem

model diet.mod;          # load the optimization model
data diet.dat;           # load the model data
options solver cplexamp; # choose cplex as optimization solver
solve;                   # solve the problem
display Buy;             # display the optimal solution
```

The status bar at the bottom of the window indicates "Ln 8, Col 1" and "INS".

The Diet Problem - AMPL implementation

- Open Terminal
- Go to the folder *diet*
- Evaluate commands in run file *diet.run* by AMPL



```
zuzana@math-pc23:~/Desktop/diet
File Edit View Terminal Tabs Help
[zuzana@math-pc23 ~]$ cd Desktop/diet/
[zuzana@math-pc23 diet]$ ampl diet.run;
```

The Diet Problem - AMPL implementation

```

zuzana@math-pc23:~/Desktop/diet
File Edit View Terminal Tabs Help
[zuzana@math-pc23 ~]$ cd Desktop/diet/
[zuzana@math-pc23 diet]$ ampl diet.run;
IBM ILOG License Manager: "university-goteborg" is accessing AMPL 12.
IBM ILOG AMPL 12.1.0 (5724-Y45) AMPL Version 20090327 (Linux 2.6.18-6-amd64)
IBM ILOG License Manager: "university-goteborg" is accessing CPLEX 12 with option(s): "b e m q use=10 ".
CPLEX 12.1.0: optimal solution; objective 143.965
4 dual simplex iterations (0 in phase I)
Buy [*] :=
BEEF  22
  CHK  5.5
  FISH 26
;

[zuzana@math-pc23 diet]$

```

The Diet Problem - AMPL implementation

- Perform sensitivity analysis
- Preserve the sensitivity analysis information
- Use suffices for sensitivity analysis: *.rc*, *.slack*, *.dual*, ...

```

# Model file for the Diet Problem

model diet.mod;          # load the optimization model
data diet.dat;           # load the model data
options solver cplexamp; # choose cplex as optimization solver

option cplex_options 'sensitivity'; # preserve the sensitivity
option presolve 0;         # analysis information
option solve;             # do not reduce the problem

solve;                   # solve the problem
display Buy;              # display the optimal solution
display Diet.dual;        # display dual variables
display Food_Max.slack;   # display slack variables
display Buy.rc;           # display reduced costs
  
```


The Diet Problem - AMPL implementation

- Change type of variables: *integer*, *binary*, ...
- Sensitivity analysis as described is possible for linear programs in continuous variables

```

diet.mod (~/Desktop/diet) - gedit
File Edit View Search Tools Documents Help
New Open Save Print... Undo Redo Cut Copy Paste Find Replace
diet.mod x
# Model file for the Diet Problem

set FOOD; # index set for foods
set NUTR; # index set for nutrients

var Buy {i in FOOD} integer >= 0; # decision variables

param cost {FOOD}; # costs of foods
param f_max {FOOD}; # available amounts of foods
param perc {FOOD,NUTR}; # percentage of nutrients in foods
param n_min; # requirement for nutrients

minimize Total_Cost: sum{i in FOOD} cost[i] * Buy[i];

subject to Diet {j in NUTR}:
    sum {i in FOOD} perc[i,j] * Buy[i] >= n_min;

subject to Food Max {i in FOOD}:

```

Ln 6, Col 29 INS

The Diet Problem - AMPL implementation

```
zuzana@math-pc23:~/Desktop/diet
File Edit View Terminal Tabs Help
[zuzana@math-pc23 ~]$ cd Desktop/diet/
[zuzana@math-pc23 diet]$ ampl diet.run;
IBM ILOG License Manager: "university-goteborg" is accessing AMPL 12.
IBM ILOG AMPL 12.1.0 (5724-Y45) AMPL Version 20090327 (Linux 2.6.18-6-amd64)
IBM ILOG License Manager: "university-goteborg" is accessing CPLEX 12 with optio
n(s): "b e m q use=10 ".
CPLEX 12.1.0: optimal integer solution; objective 144.96
2 MIP simplex iterations
0 branch-and-bound nodes
1 mixed-integer rounding cut
Buy [*] :=
BEEF 22
  CHK  5
FISH 27
;

[zuzana@math-pc23 diet]$
```

Other useful AMPL commands

- AMPL options: *option ...;*
- CPLEX options: *option cplex_options ...;*
- Define higher dimensional parameters:

$$\text{param } a := [1, *, *]: \dots := \dots$$

$$[2, *, *]: \dots := \dots;$$
- Set parameter value from run file:

$$\text{let param}[i]:= 0;$$
- Display information in terminal window:

$$\text{print "..."};$$
- $\text{if } (\dots) \text{ then } \{ \dots \} \text{ else if } (\dots) \text{ then } \{ \dots \};$
- $\text{for } \{ i \text{ in } I \} \{ \dots \};$
- $\text{break};$

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Assignment 1: Biodiesel supply chain

Below is a description of the biodiesel supply chain problem such that the total profit from supplying the demand of biodiesel is maximized. The assignment tasks are to

- formulate linear optimization model for the described problem,
- model and solve the problem using AMPL and CPLEX, and
- analyze the results and answer a number of given questions.

Study the Modeling Language for Mathematical Programming AMPL and the solver CPLEX using the following links or the recommended exercise on linear optimization and software from the course homepage before you start solving the exercises.

<http://www.ampl.com/BOOK/download.html>

<http://www.ampl.com/BOOKLETS/amplcplex122userguide.pdf>

To pass the assignment you should (in groups of two persons) give satisfactory answers to the following questions in a written report in the form of a PDF file. You should write the report on a computer, preferably using LaTeX. You shall also estimate the number of hours spent on this assignment and note this in your report. You may discuss the problem with other students. However, each group must hand in their own solution. The report will be checked for plagiarism via <http://www.urkund.com>. The questions 1, 2, and 3a)-f) are mandatory. In addition, students aiming at grade 4, 5 or VG must answer the questions 3g)-h).

The file containing your report shall be called **Name1-Name2-Ass1.pdf**, where "Name k ", $k = 1, 2$, is your respective family name. **Do not forget to write the authors' names also inside the report.** The report should be 3-4 pages long excluding illustrating diagrams and it should be

submitted in PingPong at latest Wednesday 22nd of April 2015, 23.55.

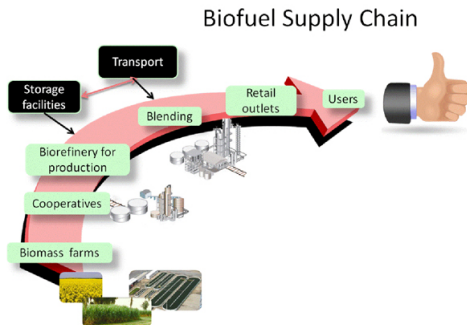
Biofuels supply chain

- Reduce oil dependence
- Reduce greenhouse effect and climate change
- Substitute fuel in transportation sector
- Biofuels can be used in existing cars
- EU quotas to use 10 % of energy from renewable sources in transportation by 2020, to use 5 % of biodiesel in diesel fuel from 2003
- Food versus fuel debate
- Develop a mathematical model of the biofuels supply chain

Biofuels supply chain

The value chain typically includes:

- Feedstock production
- Biofuel production
- Blending
- Distribution
- Consumption



Assignment 1: Biodiesel supply chain

- Biodiesel supply chain problem
- Maximize the total profit
- Supply the demand of biodiesel
- Tasks:
 - Formulate linear optimization model
 - Model and solve the problem using AMPL and CPLEX
 - Perform sensitivity analysis

Crops

- Data:
 - Available area
 - Soya, Sunflower, Cotton
 - Each crop yields expected amount of seeds
 - Each crop has water demand
 - Available water
- Processes
 - Extraction of vegetable oils from seeds (given yields)
 - Transesterification: vegetable oil + methanol = biodiesel (given proportions)
 - Purchase methanol (given price)

Final Products

- Data:
 - B5, B30, B100
 - Each product has price
 - Each product has tax (higher amount of biodiesel \Rightarrow lower tax)
 - Demand of fuels to be delivered
- Processes
 - Blending of biodiesel and petrol diesel
 - Purchase petrol diesel (given price and availability)

Sensitivity analysis






- Analyze results and answer several important questions without changing the model.
- How sensitive is the optimal solution to changes in data values?
 - *Reduced costs* of a non-basic variables: the change in the objective value when the value of the corresponding variable is (marginally) increased
 - *Shadow price* of a constraint: the change in the optimal value if you change (marginally) the RHS, it equals to the optimal value of the corresponding dual variable
 - *Slack variables* of a constraint: indicate how much the RHS can be reduced while staying feasible
- Use these tools to answer the questions

Cetane number

- Quality of pure biodiesel is given by cetane number
- Quality of petrol diesel is given by octane number
- Octane-cetane relationship is linear
- Requirement for quality of each product should be incorporated

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-  R. Fourer, D.M. Gay, and B.W. Kernighan, *AMPL: A Modeling Language for Mathematical Programming*, Duxbury Press, 2003,
<http://www.ampl.com/BOOK/download.html>.
-  *IBM ILOG AMPL, Version 12.2, User's Guide, Standard (Command-line) Version Including CPLEX Directives*, IBM, May 2010,
<http://www.ampl.com/BOOKLETS/amplcplex122userguide.pdf>.
-  Z. Šabartová, A.-B. Strömberg, *Assignment 1: Biodiesel supply chain*, March 20, 2015,
<http://www.math.chalmers.se/Math/Grundutb/CTH/mve165/1415/>.
-  Ch. Papapostolou, E. Kondili, J. K. Kaldellis, *Development and implementation of an optimisation model for biofuels supply chain*, *Energy*, Volume 36, Issue 10, October 2011, Pages 6019–6026.
-  J. Lundgren, M. Rönnqvist, P. Värbrand, *Optimization*, Studentlitteratur AB, Lund, 2010.