MVE165/MMG631
Linear and Integer Optimization with Applications
Lecture 2
AMPL and CPLEX, Assignment 1

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Overview

AMPL

- Algebraic modelling language for optimization problems
  - $\Longrightarrow$ Interface between problems and solvers
  - $\Longrightarrow$ Formulate optimization models and examine solutions
  - $\Longrightarrow$ Manage communication with an appropriate solver
- Natural syntax
- Separation of model and data
- Support for sets and set operators
- Built in arithmetic functions
- Looping, if-then-else commands

Solvers that work with AMPL

- **CPLEX** - linear and quadratic problems in continuous and integer variables
- Gurobi - linear and quadratic problems in continuous and integer variables
- CONOPT - nonlinear problems in continuous variables
- MINOS - linear and nonlinear problems in continuous variables
- CONDOR, Gecode, IPOPT, MINLP, SNOPT, etc.

Overview

CPLEX

- Optimization software package for solving linear and quadratic problems in continuous and integer variables
- Originally based on simplex method implemented in C
- Primal and dual simplex method
- Barrier method
- Techniques to avoid degeneracy
- Cutting planes
- Branch & Bound algorithm
- Heuristics

Overview

The Diet Problem - description

- G. B. Dantzig, 1990
- Choose prepared foods to meet certain nutritional requirements in the cheapest way
- Precooked foods (beef, chicken, fish) are available in given quantity and for given price
- Each food provide given percentage of daily requirements of nutrients (A, C, B1, B2)

	Price [$]	Av. [pcs]	A [%]	C [%]	B1 [%]	B2 [%]
Beef	3.19	22	60	20	10	15
Chicken	2.59	48	8	0	20	20
Fish	2.29	45	8	10	15	10

- Demand: meet week's requirements, 700 % of daily requirements for each nutrient

The Diet Problem - mathematical model

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- Sets:
  - $I = 1, 2, 3$ - set of kinds of food
  - $J = 1, 2, 3, 4$ - set of nutrients

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- Variables:
  - $x_i, \ i = 1, \ldots, 3$ - purchased amount of food i [pcs]

The Diet Problem - mathematical model

- Sets:
  - $I = 1, 2, 3$ - set of kinds of food
  - $J = 1, 2, 3, 4$ - set of nutrients
- Variables:
  - $x_i,\ i = 1, \ldots, 3$ - purchased amount of food i [pcs]
- Parameters:
  - $c_i,\ i = 1, \ldots, 3$ - cost of one piece of food i [\$]
  - $a_i,\ i = 1, \ldots, 3$ - available amount of food i [pcs]
  - $p_{ij},\ i = 1, \ldots, 3,\ j = 1, \ldots, 4$ - percentage of daily requirement of nutrient j in food i [%]
  - d - requirement for nutrients [%]

The Diet Problem - model

$$\min \quad \sum_{i=1}^{3} c_i x_i$$

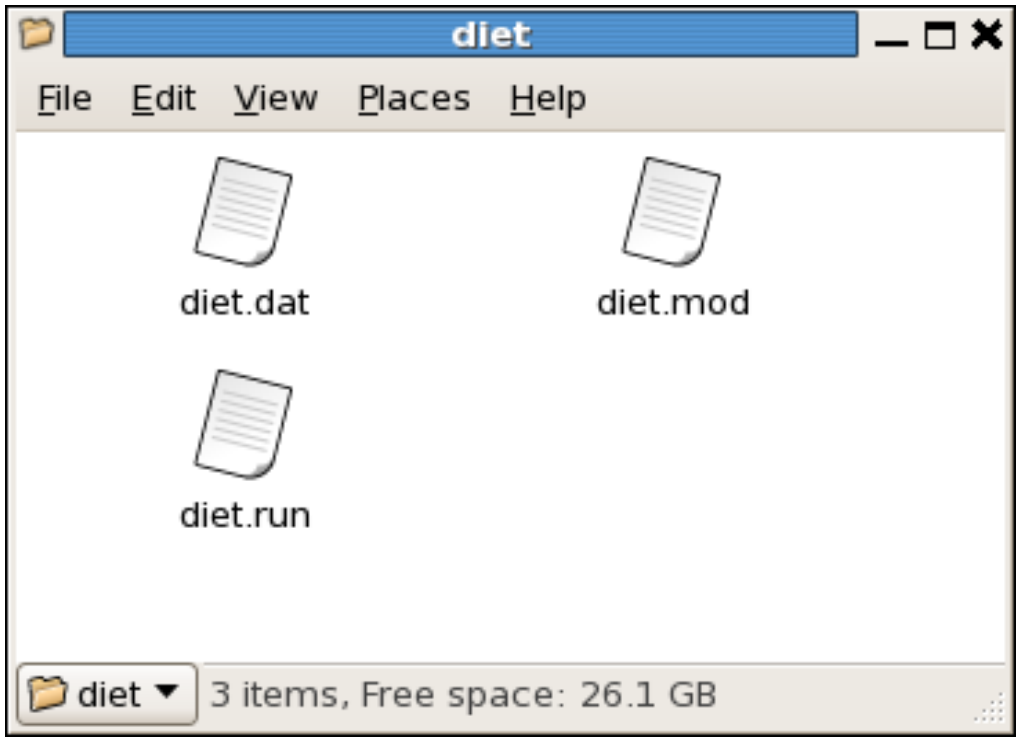
$$\text{s.t.} \quad \sum_{i=1}^{3} p_{ij}\, x_i \geq d, \quad j = 1, \ldots, 4$$

$$x_i \leq a_i, \quad i = 1, \ldots, 3$$

$$x_i \geq 0, \quad i = 1, \ldots, 3$$

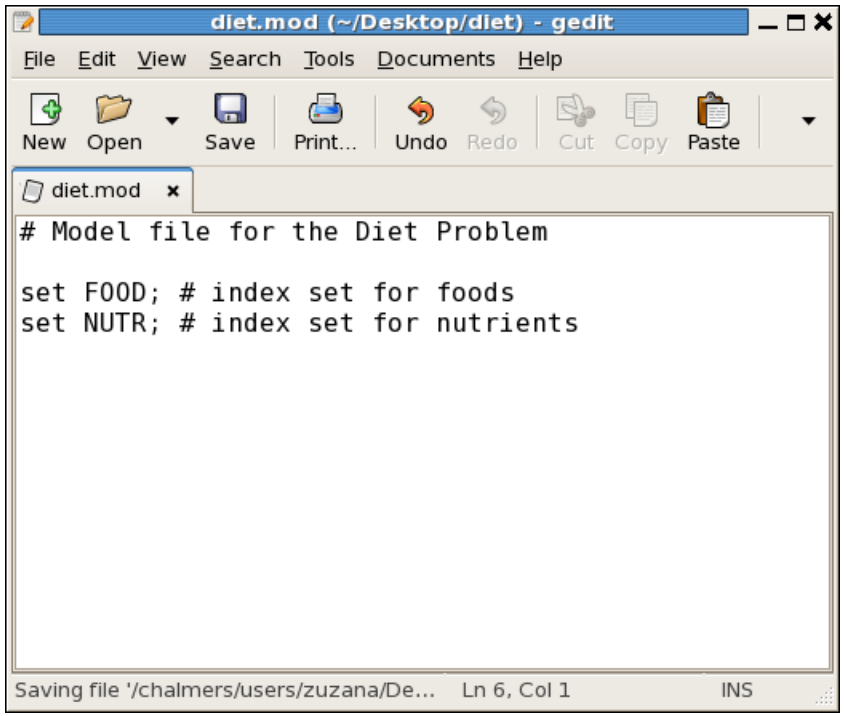
The Diet Problem - AMPL implementation

- Create folder *diet*
- Create model file *diet.mod*
- Create data file *diet.dat*
- Create run file *diet.run*



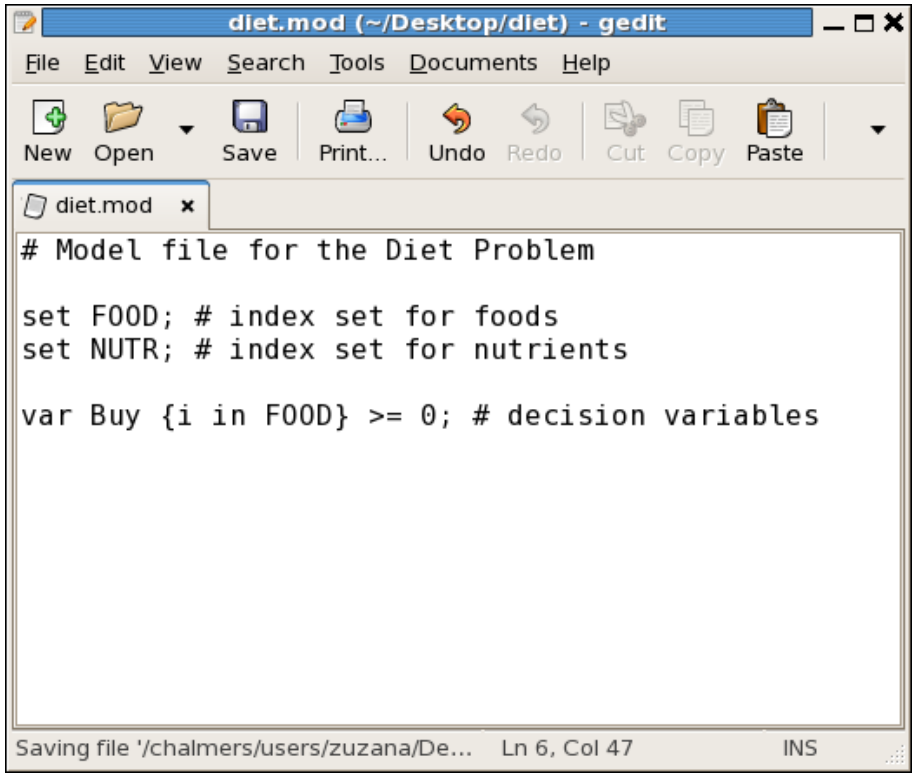
The Diet Problem - AMPL implementation

- Fill the model file using text editor (Emacs, gedit, ...)
- Introduce index sets: *set*
- Comments start with #, each command ends with ;
- Sets:
  - $I = 1, 2, 3$
  - $J = 1, 2, 3, 4$



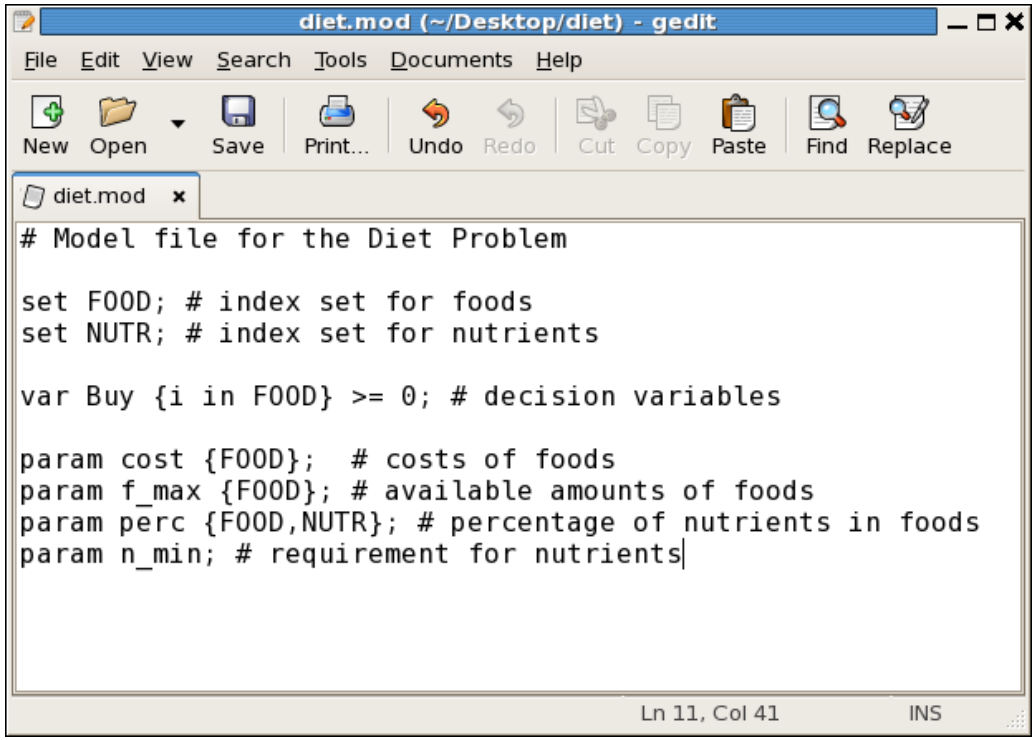
The Diet Problem - AMPL implementation

- Introduce variables: *var*
- Formulate non-negativity requirements
- Variables:
  - $x_i,\ i = 1, \ldots, 3$
  - $x_i \geq 0,\ i = 1, \ldots, 3$



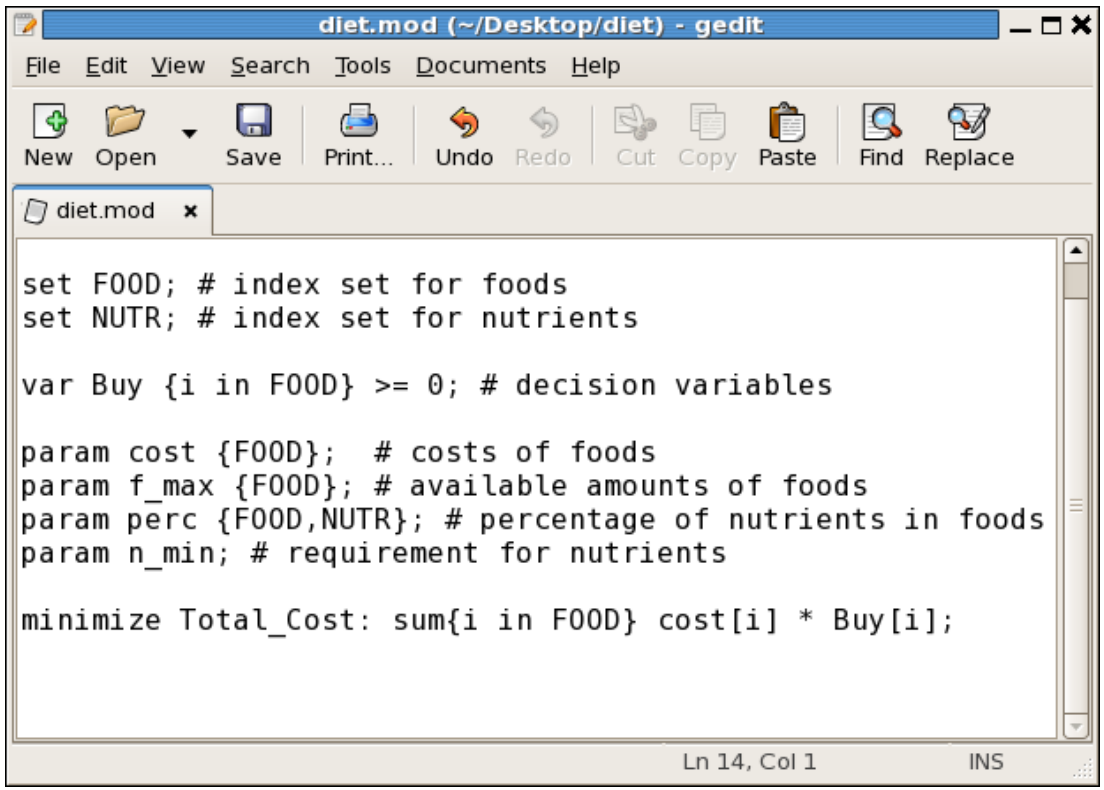
The Diet Problem - AMPL implementation

- Introduce parameters: *param*
- Parameters:
  - $c_i,\ i = 1, \ldots, 3$
  - $a_i,\ i = 1, \ldots, 3$
  - $p_{ij},\ i = 1, \ldots, 3,\ j = 1, \ldots, 4$
  - d



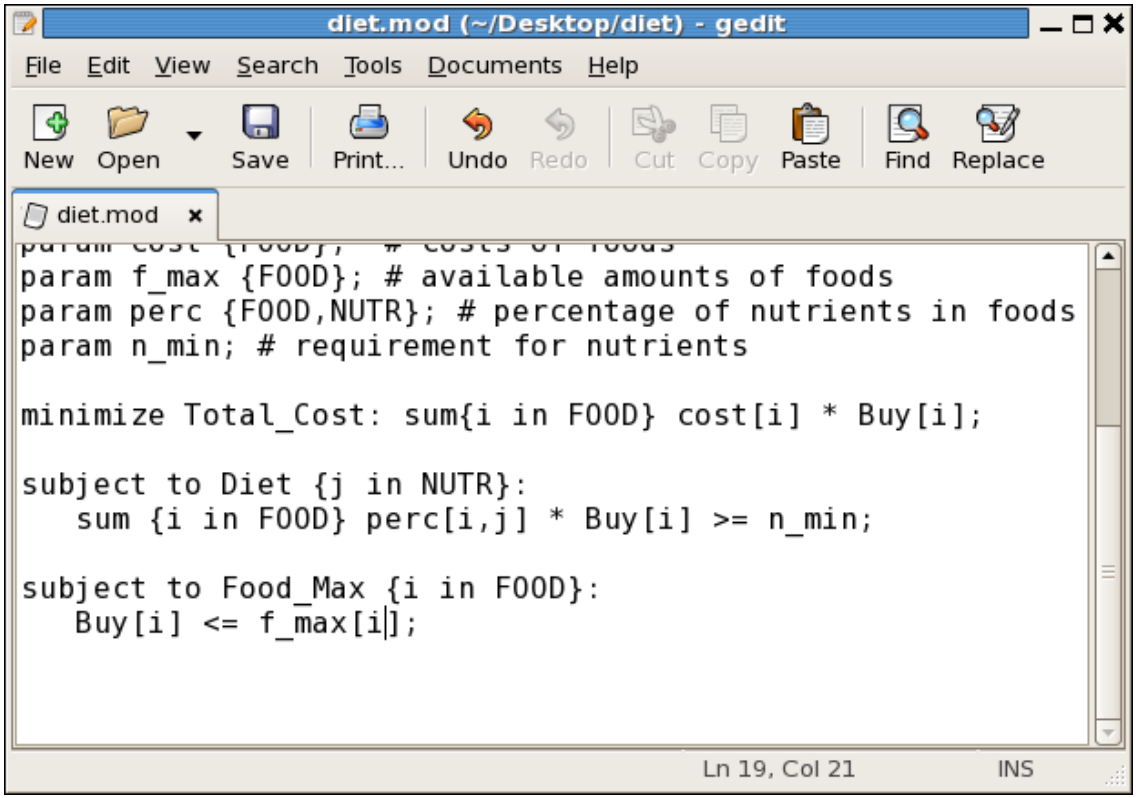
The Diet Problem - AMPL implementation

- Formulate objective function: *minimize*, *maximize*
- Use built-in arithmetic functions: $+$, $-$, $*$, $\hat{}$, $/$, *sum*, *prod*, *abs*, *log*, *sin*, ...
- min $\sum_{i=1}^{3} c_i x_i$



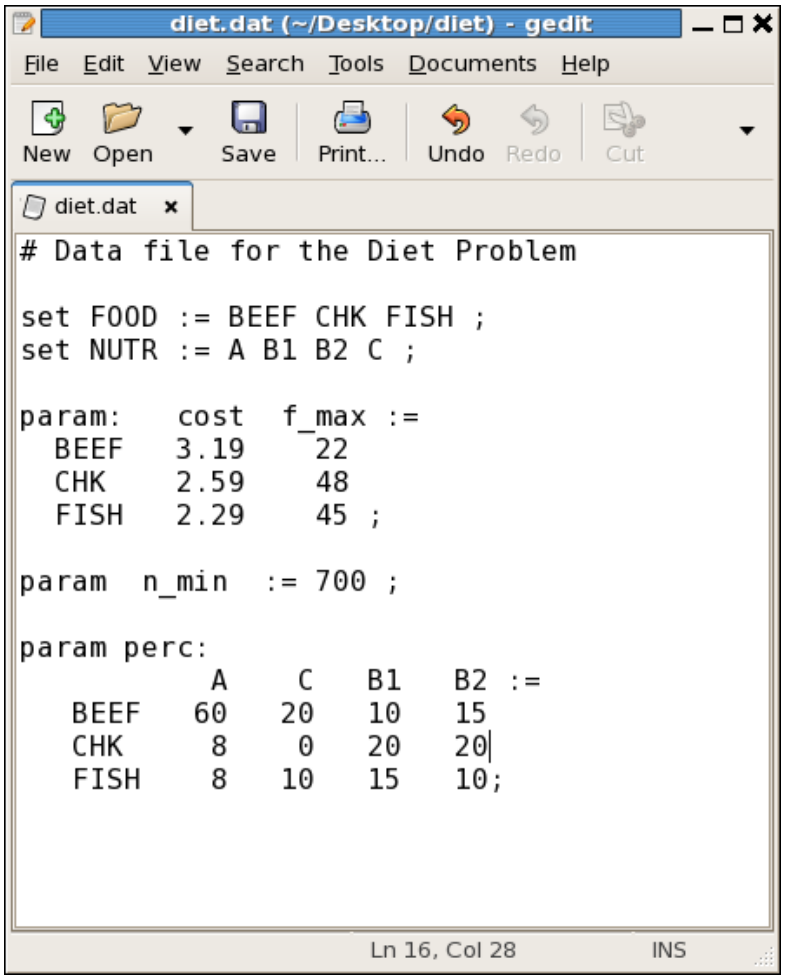
The Diet Problem - AMPL implementation

- Formulate constraints: *subject to*
- Use arithmetic relations: $>$, $>=$, $<$, $<=$, $==$, $!=$, ...
- $\sum_{i=1}^{3} p_{ij} x_i \geq d, \ j = 1, \ldots, 4$
- $x_i \leq a_i, \ i = 1, \ldots, 3$



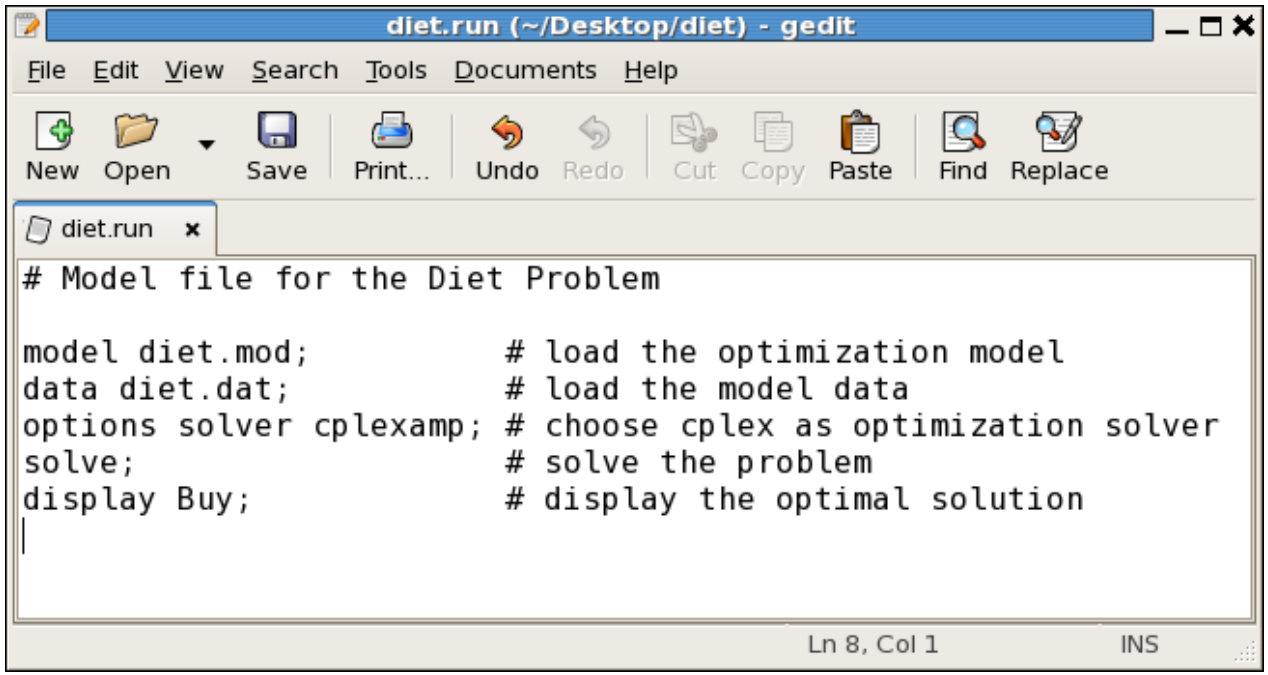
The Diet Problem - AMPL implementation

- Fill the data file using text editor
- Assign values to introduced sets and parameters



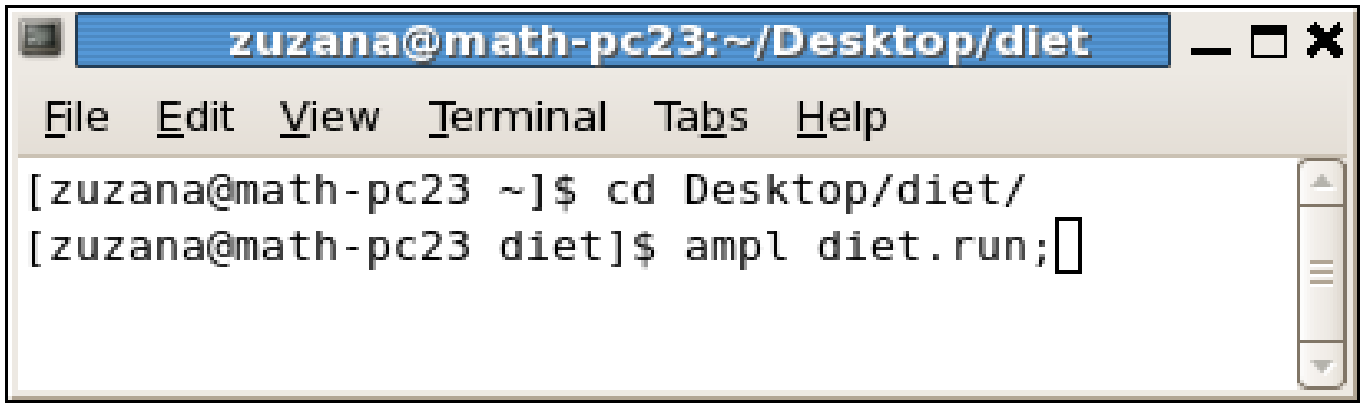
The Diet Problem - AMPL implementation

- Fill the run file using text editor
- Load the model and data: *model*, *data*
- Choose solver: *options solver*
- Solve the problem: *solve*
- Display results: *display*

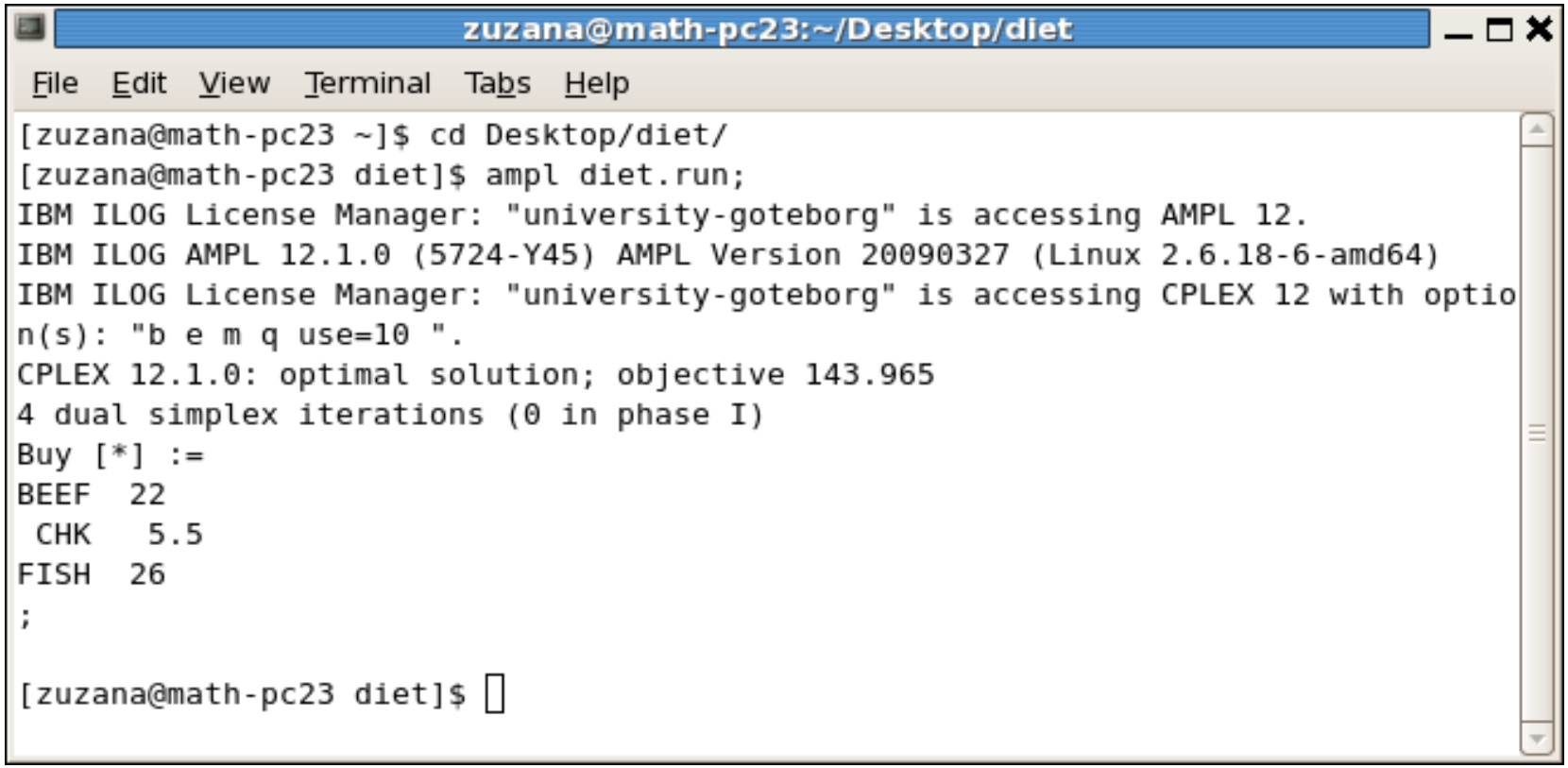


The Diet Problem - AMPL implementation

- Open Terminal
- Go to the folder *diet*
- Evaluate commands in run file *diet.run* by AMPL

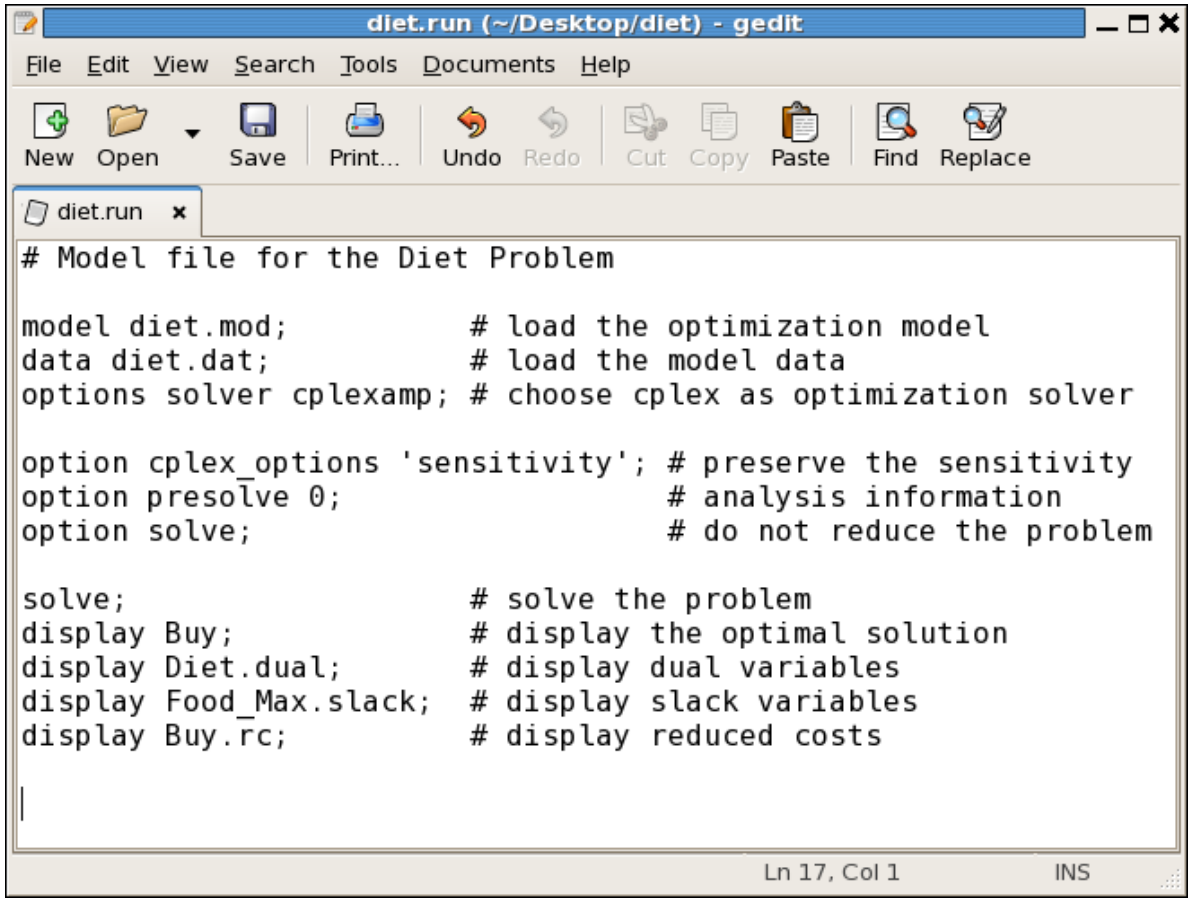


The Diet Problem - AMPL implementation



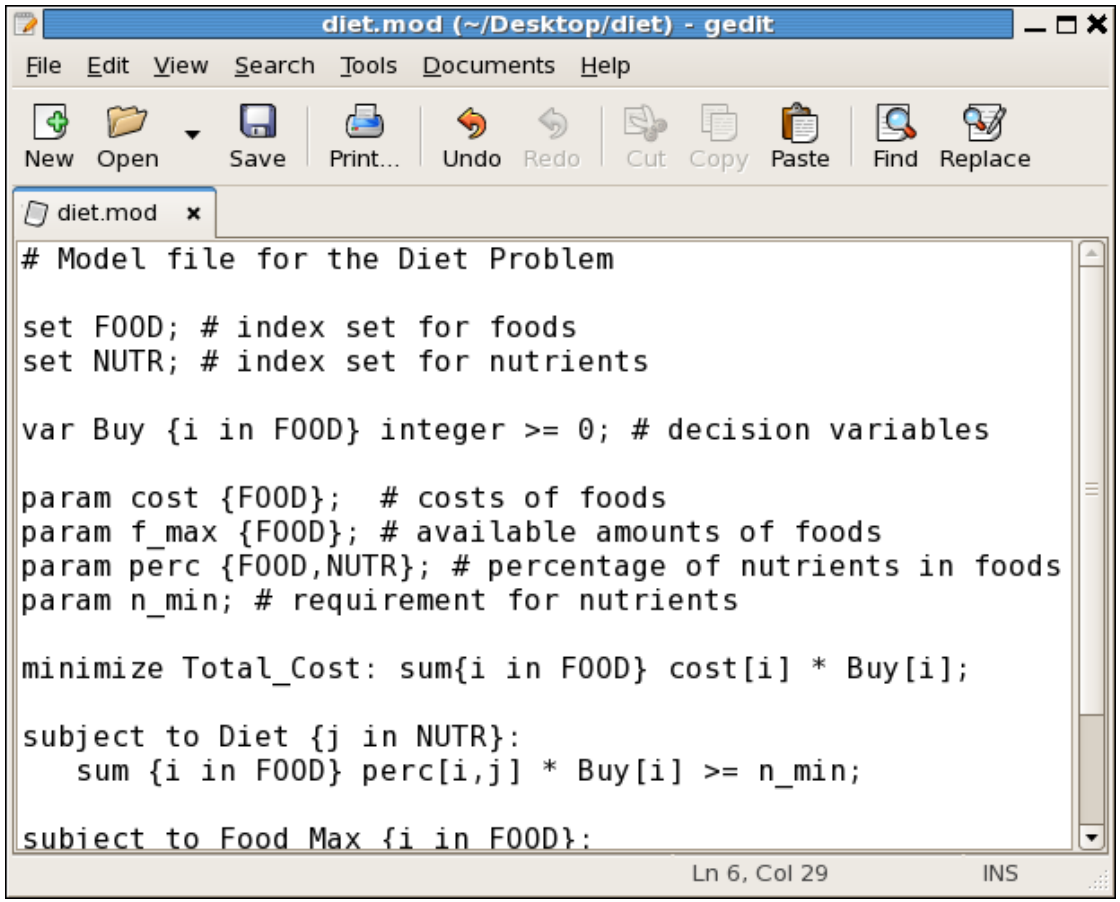
The Diet Problem - AMPL implementation

- Perform sensitivity analysis
- Preserve the sensitivity analysis information
- Use suffices for sensitivity analysis: *.rc*, *.slack*, *.dual*, ...

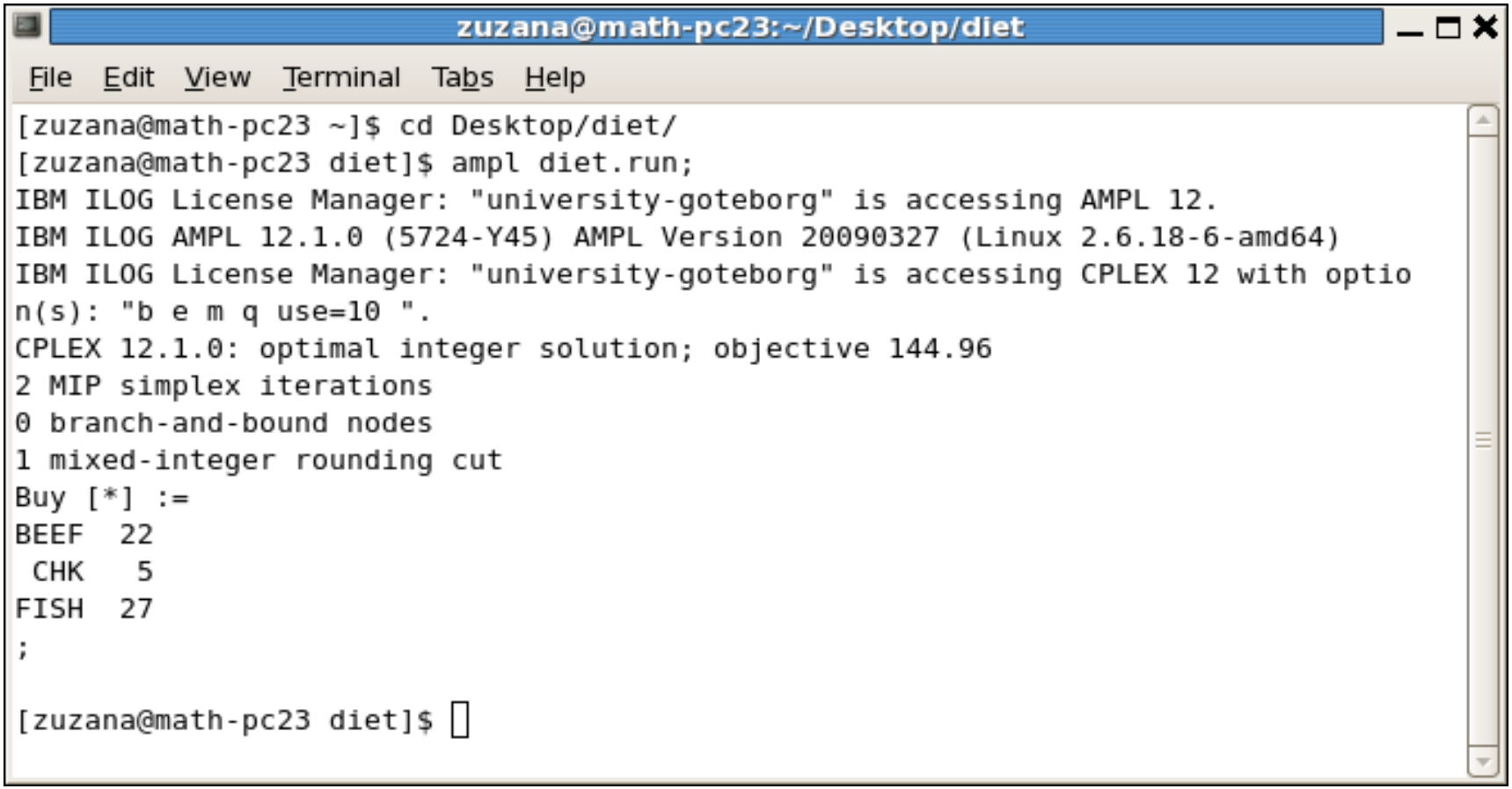


The Diet Problem - AMPL implementation

- Change type of variables: *integer*, *binary*, ...
- Sensitivity analysis as described is possible for linear programs in continuous variables



The Diet Problem - AMPL implementation



Other useful AMPL commands

- AMPL options: *option ...;*
- CPLEX options: *option cplex_options ...;*
- Define higher dimensional parameters:
  param a:= [1,,*]: ... := ...*
  [2,,*]: ... := ...;*
- Set parameter value from run file:
  let param[i]:= 0;
- Display information in terminal window:
  print "...";
- *if (...) then {...} else if (...) then {...};*
- *for {i in I} {...};*
- *break;*

Overview

Chalmers University of Technology
University of Gothenburg
Mathematical Sciences
Optimization
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MVE165
MMG631
Linear and integer optimization
with applications
Assignment information
March 20, 2015

Assignment 1: Biodiesel supply chain

Below is a description of the biodiesel supply chain problem such that the total profit from supplying the demand of biodiesel is maximized. The assignment tasks are to

- formulate linear optimization model for the described problem,
- model and solve the problem using AMPL and CPLEX, and
- analyze the results and answer a number of given questions.

Study the Modeling Language for Mathematical Programming AMPL and the solver CPLEX using the following links or the recommended exercise on linear optimization and software from the course homepage before you start solving the exercises.

`http://www.ampl.com/BOOK/download.html`
`http://www.ampl.com/BOOKLETS/amplcplex122userguide.pdf`

To pass the assignment you should (in groups of two persons) give satisfactory answers to the following questions in a written report in the form of a PDF file. You should write the report on a computer, preferably using LaTeX. You shall also estimate the number of hours spent on this assignment and note this in your report. You may discuss the problem with other students. However, each group must hand in their own solution. The report will be checked for plagiarism via `http://www.urkund.com`. The questions 1, 2, and 3a)–f) are mandatory. In addition, students aiming at grade 4, 5 or VG must answer the questions 3g)–h).

The file containing your report shall be called **Name1-Name2-Ass1.pdf**, where "Name$k$", $k = 1, 2$, is your respective family name. **Do not forget to write the authors' names also inside the report.** The report should be 3–4 pages long excluding illustrating diagrams and it should be

submitted in PingPong at latest Wednesday 22nd of April 2015, 23.55.

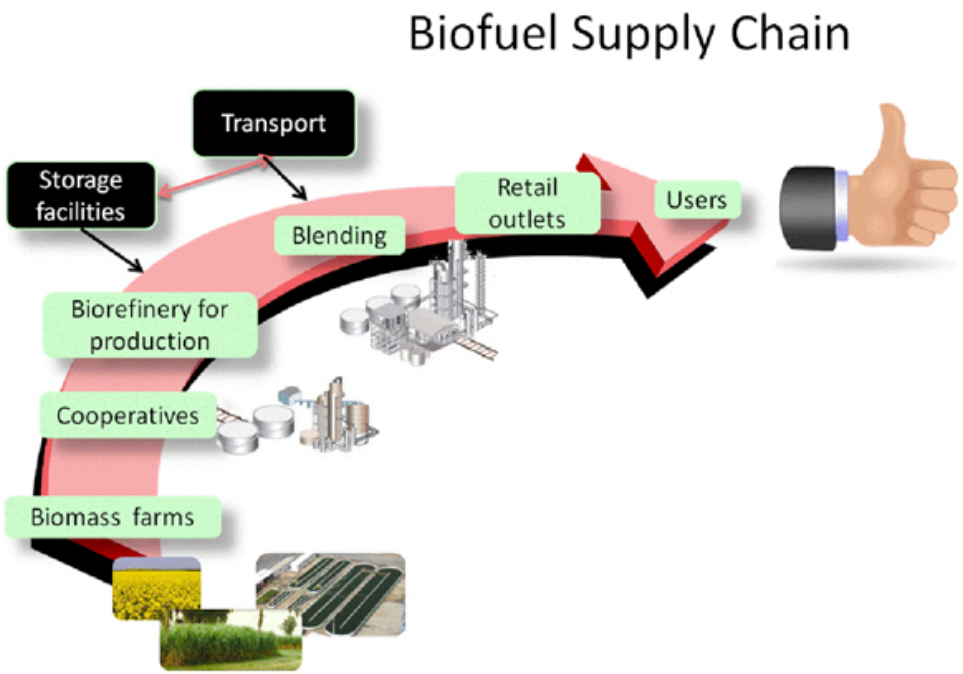
Biofuels supply chain

- Reduce oil dependence
- Reduce greenhouse effect and climate change
- Substitute fuel in transportation sector
- Biofuels can be used in existing cars
- EU quotas to use 10 % of energy from renewable sources in transportation by 2020, to use 5 % of biodiesel in diesel fuel from 2003
- Food versus fuel debate
- Develop a mathematical model of the biofuels supply chain

Biofuels supply chain

The value chain typically includes:

- Feedstock production
- Biofuel production
- Blending
- Distribution
- Consumption



Assignment 1: Biodiesel supply chain

- Biodiesel supply chain problem
- Maximize the total profit
- Supply the demand of biodiesel
- Tasks:
  - Formulate linear optimization model
  - Model and solve the problem using AMPL and CPLEX
  - Perform sensitivity analysis

Crops

- Data:
  - Available area
  - Soya, Sunflower, Cotton
  - Each crop yields expected amount of seeds
  - Each crop has water demand
  - Available water
- Processes
  - Extraction of vegetable oils from seeds (given yields)
  - Transesterification: vegetable oil + methanol = biodiesel (given proportions)
  - Purchase methanol (given price)

Final Products

- Data:
  - B5, B30, B100
  - Each product has price
  - Each product has tax (higher amount of biodiesel $\Rightarrow$ lower tax)
  - Demand of fuels to be delivered
- Processes
  - Blending of biodiesel and petrol diesel
  - Purchase petrol diesel (given price and availability)

Sensitivity analysis

- Analyze results and answer several important questions without changing the model.
- How sensitive is the optimal solution to changes in data values?
  - *Reduced costs* of a non-basic variables: the change in the objective value when the value of the corresponding variable is (marginally) increased
  - *Shadow price* of a constraint: the change in the optimal value if you change (marginally) the RHS, it equals to the optimal value of the corresponding dual variable
  - *Slack variables* of a constraint: indicate how much the RHS can be reduced while staying feasible
- Use these tools to answer the questions

Cetane number

- Quality of pure biodiesel is given by cetane number
- Quality of petrol diesel is given by octane number
- Octane-cetane relationship is linear
- Requirement for quality of each product should be incorporated

Overview


R. Fourer, D.M. Gay, and B.W. Kernighan, *AMPL: A Modeling Language for Mathematical Programming*, Duxbury Press, 2003, `http://www.ampl.com/BOOK/download.html`.


IBM ILOG AMPL, Version 12.2, User's Guide, Standard (Command-line) Version Including CPLEX Directives, IBM, May 2010, `http://www.ampl.com/BOOKLETS/amplcplex122userguide.pdf`.


Z. Šabartová, A.-B. Strömberg, *Assignment 1: Biodiesel supply chain*, March 20, 2015, `http://www.math.chalmers.se/Math/Grundutb/CTH/mve165/1415/`.


Ch. Papapostolou, E. Kondili, J. K. Kaldellis, *Development and implementation of an optimisation model for biofuels supply chain*, Energy, Volume 36, Issue 10, October 2011, Pages 6019–6026.


J. Lundgren, M. Rönnqvist, P. Värbrand, *Optimization*, Studentlitteratur AB, Lund, 2010.