Chalmers University of Technology
University of Gothenburg
Mathematical Sciences
Optimization
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MVE165
MMG631
Linear and integer optimization
with applications
Assignment information
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Assignment 3b:
The Scandinavian electricity system

Given below is a description of the problem to design a new Scandinavian electricity system. The assignment tasks are to `(a)` formulate the problem(s) using mixed integer linear optimization, `(b)` model and solve them using AMPL and CPLEX, and `(c)` analyze the results and answer a number of questions below. Material for the assignment is found at the course homepage: `www.math.chalmers.se/Math/Grundutb/CTH/mve165/1617/`

To pass the assignment you should (in groups of two persons) `(i)` write a report (maximum six pages excluding figures/illustrations) that describes and discusses the issues presented in the exercises and questions below. You shall also estimate the number of hours spent on this assignment and note this in your report.

The file containing your report shall be called **Name1-Name2-Ass2.pdf**, where "Name*k*", $k = 1, 2$, is your respective family name. **Do not forget to write the authors' names also inside the report.**

The report should be **submitted in PingPong at latest Thursday 18th of May 2017.**

You shall also `(ii)` present your assignment orally at a seminar on **May 19, 22, 23, or 24, 2017.** The seminars are scheduled via a doodle link from the course home page. Presence is mandatory at at least one full seminar.

1 Problem background

In this assignment, a new electricity system in Scandinavia, e.g. Sweden, Norway and Denmark, should be designed. An electricity system is composed of several different generation technologies. Each country defines a region, and it is assumed that the current hydropower capacity installment in the regions is fixed. New capacity investments are, however, possible in wind, nuclear, and fossil fuel (i.e. coal and natural gas) power plants. Moreover, each region only considers aggregated continuous capacity for each of the different generation technologies. This means that, for each region and technology type, all power plants are combined. Thus, you may model this as one giant power plant per technology type in each region.

Note that the concept *capacity* here denotes the maximum possible electricity output over a certain amount of time, say τ [h]. If the capacity is, for example, 5 MW, the maximum electricity output during the time period τ hours is 5τ MWh.

Investments should be minimized but sufficient to cover the electricity demand; see Table 1. Two normalized profiles (`elGen_demand8760.dat`, 1 h resolution, and `elGen_demand2920.dat`, 3 h resolution), in which each data value represents a share of the total demand in a region at a specific time interval during the year, can be found on the course homepage.

The regions, as noted in the table, have their own demands, but also their own electricity production. It is, however, possible to freely trade between the regions, although with a transmission loss of 10% of the traded electricity. The system is furthermore isolated, which means that no external trade is allowed. Figure 1 illustrates the Scandinavian transmission system.

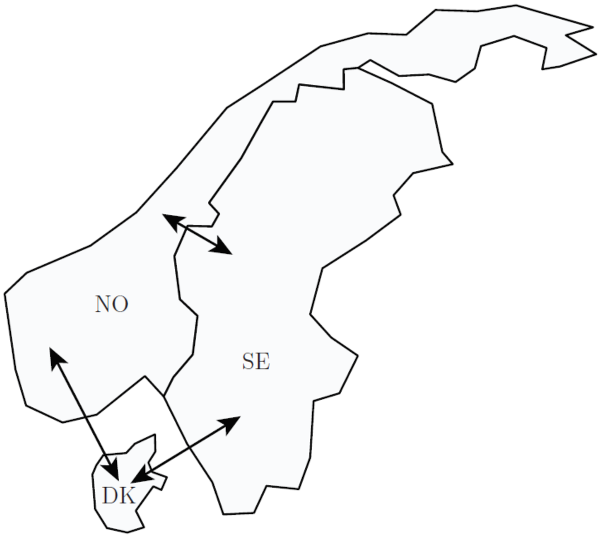

Figure 1: Illustration of the transmission possibilities

Region	Annual demand [TWh_{el}]
SE	147.2
NO	131.8
DK	35.5

Table 1: The annual electricity demand for each region

To be able to generate electricity, proper investments in different power plants are required to take place beforehand. However, the problem with making investments is that only the *present value* of the investment is known. Thus, to account for interest rates and the life spans of the respective technologies, the annuity payment factor a_n of each investment should be used; it is defined as

$$a_n = \frac{r}{1 - (1 + r)^{-n}}, \tag{1}$$

where r [%] denotes the interest rate and n [years] denotes the life span of the investment.[1]

Some additional necessary properties for the different technologies and their respective fuel types are listed in Tables 2 and 3. Each technology requires a unique fuel type (except for hydro- and wind power, which are fuelless), for which the transformation into electricity has a specific efficiency. Moreover, varying operation and maintenance (O&M) costs are also added, depending on electricity generation scheme.

Technology specific attributes are given in the following subsections.

Energy type	Fuel	Efficiency [%]	Life span [years]	Investment costs [€/MW]	O&M costs [€/MWh_{el}]	Emissions [$\text{gCO}_2/\text{kWh}_{\text{el}}$]
Nuclear (L)	uranium	0.33	60	6.1	9.8	115
Nuclear (S)	uranium	0.33	60	6.1	9.8	115
Hydro	–	1	–	–	1	0
Wind	–	1	25	1.7	7.5	0
Coal	coal	0.39	50	1.6	3.6	1000
Gas	natural gas	0.40	20	0.47	2.3	350

Table 2: Plant properties

Fuel	Costs [€/MWh_{fuel}]
uranium	4.8
coal	6.5
natural gas	19.7

Table 3: Fuel properties

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[1] Formula (1) is the result of a geometric series

1.1 Hydropower

The current installed capacity of the hydropower in the Scandinavian electricity system is found in Sweden and Norway, while Denmark has none. The inflow to the reservoirs depends on weather and climatic properties. Two normalized profiles (`elGen_hydro8760.dat`, 1 h resolution, and `elGen_hydro2920.dat`, 3 h resolution) of this inflow, as share of the total yearly inflow at a specific time interval, are found on the course homepage. Electricity is then produced by leading the water through turbines. The power extracted from the water depends on the volume and on the difference in height between the water's in- and outflow. Figure 2 provides an illustration of this.

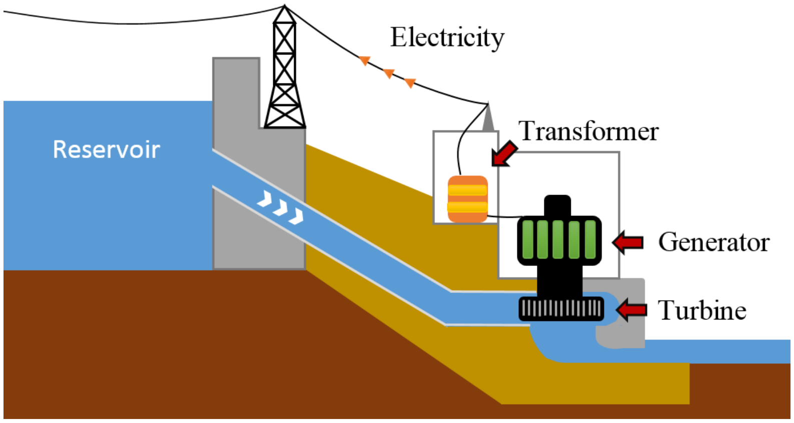

Figure 2: Hydropower electricity generation process

Moreover, assume that the reservoir level at a specific momentary time is measured in $\mathrm{MWh_{el}}/\tau$, where τ [h] is the length of the time step. Furthermore assume that the level for the hydro reservoirs when the planning period starts can be any (feasible) value, but it is required to be the same level when the planning periods ends.

Relevant data for the different regions can be found in Table 4.

Region	Installed capacity [GW]	Total yearly inflow [TWh]
SE	16.2	44
NO	30.6	89
DK	-	-

Table 4: Hydropower properties

1.2 Wind power

Wind power output is intermittent and has a significant variation over shorter time horizons. Two wind profiles are found on the course homepage (`elGen_wind8760.dat`, 1 h resolution, and `elGen_wind2920.dat`, 3 h resolution), measured as share of installed capacity, which works as an upper limit for the electricity generation each time step.

Not all areas are suitable for installment of wind farms since wind speed and terrain varies across the regions. Thus, for reasons regarding land exploitation, there is an upper limit of the wind power capacity that is available for installment in each region; see Table 5.

Region	Upper capacity limit of wind power [GW]
SE	7
NO	3
DK	10

Table 5: Wind power properties

1.3 Nuclear power

It is possible to invest into nuclear power in the system. Two different reactor types can be used: small and large, and they have the exact same properties besides their capacity size. Several reactors of each type can be bought, and their respective capacity is given in Table 6.

Reactor type	Capacity [MW]
Small	500
Large	850

Table 6: Nuclear power properties

There are some physical complications associated with large short-term variations in output from nuclear power, due to the complexity of turning reactors on and off. Furthermore, it is also very costly to do so. Therefore, to account for these limitations, we assume that the aggregated amount of nuclear power produced in each region must not be less than 80% of the installed capacity.

Exercises to perform and questions to answer

1. Formulate a mixed integer linear programming network flow model that seeks to minimize investment and running costs for electricity production in the Scandinavian electricity system, provided that demand must be met at all times. The model should cover an entire year, which can be assumed to consist of 8760 hours. The model should work regardless of the chosen length of the time step (although in integer multiples of hours). Assume at this point that there are no penalties for emissions, and that the interest rate for investments is 5%.

   Data files containing the profiles for hydropower, wind power and demand are found on the course homepage.

2. Implement the model from 1. in AMPL and solve it using CPLEX, for the following cases:

   (a) The time step length is 1 hour.
   (b) The time step length is 3 hours.

   Present your result and findings—including graphical illustrations of the total annual electricity generation for the different technology types—and discuss and motivate the differences between the two cases. Especially comment on the trade result and relate it to basic feasible solutions in the simplex method. Comment also on the CPU time and the number of variables and constraints needed to solve these instances.

3. Discuss, according to the two questions below, how to improve the model from 1. to give a more realistic result. These improvements need not be implemented, but mathematical descriptions of the new parameters, variables and constraints should be included. Note that the model should still be mixed integer linear!

   (a) Assume that the aggregated nuclear power in each region does not have to be above 80% of installed capacity at all times. Instead, add the possibility of having everything simultaneously shut down. In other words, the generation from nuclear power in each region should at all times be either 0 or above 80% of installed capacity.
   (b) Assume that there is a limit on how fast the generation can change over time in the coal-, gas- and nuclear power plants. Formulate these constraints for the cases when
      i. the limit is defined over consecutive time steps, and
      ii. the limit is defined over the diurnal cycle.

4. Adjust your mathematical model from 1. to include an upper limit, $e^{\max}$, on the total annual emissions. Reasonable values for this limit are $45 \cdot 10^6 \leq e^{\max} \leq 65 \cdot 10^6$ tonnes $\mathrm{CO_2/MWh_{el}}$.

5. Implement the model from 4. in AMPL and solve it using CPLEX. Assume a 3 hour time step. Present your result and findings. What happen to the solution times, and why?

   To reduce solution times, you can put an upper limit on the elapsed time. Thus add the following to your run-file:

   ```
   option cplex_options 'timelimit=r';
   ```

   where `r` is chosen appropriately.

6. Assuming a time step of 3 hours, consider the multi-objective optimization problem to minimize total costs and emissions. Then, the corresponding solutions from exercises 2(b) and 5. define points on the corresponding Pareto front. Construct a graph showing a number of (fairly spread) points on the Pareto front.

   As exercise 5. showed, adding constraints for the emissions to the model increases the solution times severely. Therefore use the *weighted sums method* (simultaneously minimize system costs and emissions under varying weights) to get Pareto points. For ease of interpretation, the costs should be presented in M€ and the emissions in tonnes $\mathrm{CO_2/MWh_{el}}$. Note that since the model is mixed-integer, the front may be discontinuous.