Chalmers University of Technology
University of Gothenburg
Mathematical Sciences
Optimization
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MVE165
MMG631
Linear and integer optimization
with applications
Assignment information, May 2, 2017

Assignment 3a: Windpower investment and generation

Below is a description of the problem to locate and operate a number of wind mills in an offshore wind farm. The assignment tasks are to `(a)` formulate the problem(s) using mixed integer linear optimization, `(b)` model and solve them using AMPL and CPLEX (or, e.g., Matlab and any MILP solver; see *Software → Linear optimization and software → Computer exercise* on the course homepage), and `(c)` analyze the results and answer a number of questions given below. Material for the assignment is found at the course homepage: `www.math.chalmers.se/Math/Grundutb/CTH/mve165/1617/`

To pass the assignment you should (in groups of two persons) `(i)` write a **detailed report** that gives satisfactory answers and explanations to the questions. You shall also estimate the number of hours spent on this assignment and note this in your report.

The file containing your report shall be called **Name1-Name2-Ass3a.pdf**, where "Name$k$", $k = 1, 2$, is your respective family name. **Do not forget to write the authors' names also inside the report.**

The report should be **submitted in PingPong at the latest Thursday 18th of May 2017.**

You shall also `(ii)` present your assignment orally at a seminar on **May 19, 22, 23, or 24, 2017.** The seminars are scheduled via a doodle link from the course home page. Presence is mandatory at at least one full seminar.

Problem background

A number of geographical locations for placing offshore wind turbines are given. These locations are relatively close to each other, so that the group of wind turbines can be regarded as a wind farm. A number of the locations may be chosen for placing wind turbines and with each location chosen is associated an investment cost of 34.5 MSEK. The possible locations form two distinct groups and there is an exploitation cost of 6 MSEK associated with each of these groups. See Figure 1.

There is also an option to choose between long, medium, and short blades of the turbines. The longer blades yields a possibly larger energy production, but they are also exposed to higher stresses and are associated with a higher

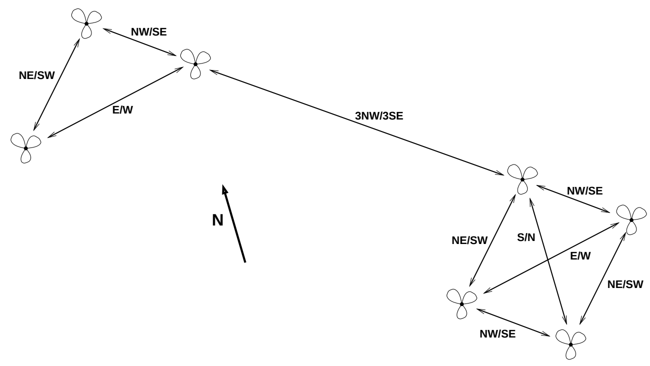

Figure 1: Map of the tentative locations for wind turbines forming two groups of turbines. The pairs of turbine locations that may be influenced by each others wind wakes (see Table 2) are indicated by arrows.

purchase cost. The purchase cost for long ($R = 45$m), medium ($R = 30$m), and short ($R = 22.5$m) blades are 2.5, 1.9, and 1.4 MSEK, respectively.

The theoretical effect, P [W], of a wind turbine is given by the expression

$$P = \frac{1}{2}\pi R^2 \varrho C_p(\lambda) v^3,$$

where R [m] denotes the radius of the turbine (i.e., the length of each turbine blade), $\varrho = 1.25$ is the density of air [kg/m^3], depends also on pressure and temperature). Further, $C_p(\lambda)$ denotes the efficiency coefficient (which cannot exceed ≈ 0.59), where λ is the tip speed ratio defined as $\lambda = \omega R/v$, and where ω denotes angular speed [rad/s] and v denotes the wind speed through the turbine [m/s].

The observed energy production and efficiency (at undisturbed wind) for different levels of wind speed and blade dimensions are listed in Table 1 together with the (discretized) wind speed distribution. The frequencies of and the corresponding mean wind speeds in different wind directions are listed in Table 2. For each of these wind directions the wind speed is assumed to be Weibull distributed with a shape parameter value of $\beta \approx 2.4$ and a scale parameter value of $\alpha \approx 0.8v$ m/s, where v denotes the mean wind speed. The Weibull cumulative distribution function for a random variable X is defined as

$$F(x; \alpha, \beta) = \begin{cases} 1 - e^{-(x/\alpha)^\beta}, & x \geq 0, \\ 0, & x < 0, \end{cases}$$

i.e., $F(x; \alpha, \beta)$ denotes the probability of $X \leq x$.

The wind turbines shadow each other through the so called wake effect. The reduction of wind and energy production due to the wake effect is dependent on the distance between the wind turbines, the wind speed, and the wind direction. The angle of the turbine blades can also be adjusted so that the energy production by this turbine is reduced but more wind is passed to the turbine(s) behind (those located within its wake). The relative power levels due to wake effects for different wind directions and relative locations of turbines are listed in Table 2.

Wind speed	Observed frequency	Observed energy production at undisturbed wind					
		Large		Medium		Small	
		effect	efficiency	effect	efficiency	effect	efficiency
[m/s]	[%]	[kW]	[%]	[kW]	[%]	[kW]	[%]
0–1	0.53	–	–	–	–	–	–
1–2	2.25	–	–	–	–	–	–
2–3	4.06	–	–	–	–	–	–
3–4	5.46	60	0.35	30		15	
4–5	7.52	140	0.39	60		35	
5–6	8.54	270	0.41	120		65	
6–7	9.46	450	0.41	190		110	
7–8	10.06	730	0.43	320		180	
8–9	10.37	1090	0.45	480		270	
9–10	9.67	1540	0.45	680		330	
10–11	8.47	1980	0.43	880		490	
11–12	6.97	2240	0.37	1100		650	
12–13	5.43	2300	0.30	1460		810	
13–14	4.07	2300	0.24	1930		1250	
14–15	2.93	2300	0.19	2210		1310	
15–16	1.90	2300	0.16	2270		1640	
16–17	1.04	2300	0.13	2300		2020	
17–18	0.57	2300	0.11	2300		2230	
18–19	0.32	2300	0.09	2300		2280	
19–20	0.16	2300	0.08	2300		2300	
20–21	0.09	2300	0.07	2300		2300	
21–22	0.05	–	–	–	–	–	–
22–23	0.02	–	–	–	–	–	–
23–24	0.03	–	–	–	–	–	–
24–25	0.01	–	–	–	–	–	–

Table 1: Observed wind speed distribution at 65m height, observed power at free wind for large (R = 45m), medium (R = 30m), and small (R = 22.5m) blades, and the corresponding efficiency (the ratio between the energy extracted and the energy content in the wind passing through the turbine).

The (theoretical) wind speed $v(x)$ in a wind wake, from the turbine and downstream, is given by the expression

$$v(x) = u \left(1 - \left(1 - \sqrt{1 - C_T}\right) \left(\frac{R}{R + \alpha x}\right)^2\right),$$

where u [m/s] denotes the speed of undisturbed wind, C_T is the so called thrust coefficient, x [m] denotes the distance downstream from the turbine, and α denotes the wake constant (the slope of the spread of the wake, onshore: $\alpha = 0.075$; offshore: $\alpha = 0.04$).

The yearly average electricity price has varied between 108 and 506 SEK/MWh since 1996, and the average over the last fifteen years is 274 SEK/MWh.

Exercises to perform and questions to answer

1. Formulate a mixed integer linear programming model that seeks to maximize the average revenue from energy production, provided that the number of installed turbines does not exceed n (a positive integer). Assume at this point that the investment cost is not bounded by any budget and that the price of electricity is constant over time. The result from the model shall describe which locations to choose for placing turbines, and which blade dimensions to choose for each of the chosen locations. The model shall also take the wake effects into account—observed wake effects in "all" wind directions are listed in Table 2 (see also Figure 1 for the interpretation of the direction notation NW, SE, etc.). Assume that multiple wake effects do *not* superpose and that we are able to control the turbines so that they always produce as much energy as possible with respect to current wind conditions (i.e., the observed effect listed in Table 1). In order to estimate the energy production you have to encounter for both the wind directions and the wind speed distribution (Weibull, see above) in each of these directions.
2. Implement the model from 1.—for all $n \in \{3, 4, 5\}$—in AMPL and solve it using CPLEX, for the following three cases:
   (a) Only the medium blade dimension ($R = 30$m) is available.
   (b) All three blade dimensions are available, but in each of the two groups of locations, only one dimension is allowed.
   (c) Any turbine may be equipped with any blade dimension.

   Present your results and findings. How large are the differences in energy production, comparing for $n = 3$, 4, and 5? Also, compare these results with those from the corresponding cases of no wake effects (which are, of course, unrealistic). Comment also on the CPU time needed to solve these instances. Can any of these instances be solved to optimality, or do you need to terminate CPLEX before an optimal solution is verified? Relate the size of the optimality gap to the CPU time used.
3. Adjust/extend your mathematical model from 1. to minimize the investment costs, provided that the resulting energy production may not be lower than that resulting from the best solution found in 2(a) for $n = 4$.
4. Implement the model from 3. in AMPL and solve it using CPLEX. Assume a suitable (with respect to computing time needed) freedom of choice for the blade dimensions (according to the cases 2(a)–2(c)). Present your results and findings. Relate the size of the optimality gap to the CPU time required.
5. Assume that all blade dimensions are available (case (c), above). The corresponding solutions from 2(c) and 4(c) define points on the corresponding Pareto front. Construct a graph showing a number of (fairly spread) points on the Pareto front; use, e.g., the ε-constraint method (maximize the production revenue under varying constraints on the investments, or minimize the investments under varying constraints on the production revenue). Since the model is mixed-binary, the Pareto front may be discontinuous. Discuss the appearance of the Pareto front for different levels of the electricity price compared to the investment costs.

Wind direction	Relative power level at a location relative to the location of an operating turbine with long blades										Frequency	Mean wind speed
[°]	N	NE	E	SE	3SE	S	SW	W	NW	3NW	[%]	[m/s]
0–10	1	1	1	1	1	0.72	1	1	1	1	1.70	6.4
10–20	1	1	1	1	1	1	0.98	1	1	1	1.88	6.1
20–30	1	1	1	1	1	1	0.81	1	1	1	0.86	6.7
30–40	1	1	1	1	1	1	0.42	1	1	1	1.32	7.4
40–50	1	1	1	1	1	1	0.58	1	1	1	1.75	7.8
50–60	1	1	1	1	1	1	0.97	1	1	1	1.85	7.0
60–70	1	1	1	1	1	1	1	0.97	1	1	1.79	6.5
70–80	1	1	1	1	1	1	1	0.96	1	1	1.79	6.6
80–90	1	1	1	1	1	1	1	0.52	1	1	1.95	7.2
90–100	1	1	1	1	1	1	1	0.72	1	1	2.27	8.6
100–110	1	1	1	1	1	1	1	0.98	1	1	2.64	8.2
110–120	1	1	1	1	1	1	1	1	0.86	0.99	2.95	8.7
120–130	1	1	1	1	1	1	1	1	0.41	0.80	3.09	8.8
130–140	1	1	1	1	1	1	1	1	0.29	0.76	3.10	8.6
140–150	1	1	1	1	1	1	1	1	0.88	0.98	2.95	9.0
150–160	1	1	1	1	1	1	1	1	0.95	1	2.73	7.9
160–170	0.96	1	1	1	1	1	1	1	1	1	2.55	7.9
170–180	0.52	1	1	1	1	1	1	1	1	1	2.50	8.2
180–190	0.72	1	1	1	1	1	1	1	1	1	2.61	8.5
190–200	1	0.98	1	1	1	1	1	1	1	1	2.91	8.3
200–210	1	0.81	1	1	1	1	1	1	1	1	3.33	8.5
210–220	1	0.42	1	1	1	1	1	1	1	1	3.80	9.2
220–230	1	0.58	1	1	1	1	1	1	1	1	4.24	9.4
230–240	1	0.98	1	1	1	1	1	1	1	1	4.58	8.9
240–250	1	0.99	1	1	1	1	1	1	1	1	4.78	9.3
250–260	1	1	0.96	1	1	1	1	1	1	1	4.86	9.2
260–270	1	1	0.52	1	1	1	1	1	1	1	4.86	9.0
270–280	1	1	0.72	1	1	1	1	1	1	1	4.80	8.8
280–290	1	1	0.90	1	1	1	1	1	1	1	4.65	8.2
290–300	1	1	1	0.86	0.99	1	1	1	1	1	4.28	8.3
300–310	1	1	1	0.41	0.80	1	1	1	1	1	3.46	6.5
310–320	1	1	1	0.29	0.76	1	1	1	1	1	1.99	6.6
320–330	1	1	1	0.88	0.98	1	1	1	1	1	1.02	7.0
330–340	1	1	1	0.94	1	1	1	1	1	1	1.20	7.7
340–350	1	1	1	1	1	0.96	1	1	1	1	1.60	7.0
350–360	1	1	1	1	1	0.52	1	1	1	1	1.36	6.9

Table 2: Wake effects of a wind mill with long blades for different wind directions, and relative locations of and distances between wind turbines, and wind direction distribution and corresponding mean wind speeds. The wind speed in each direction is assumed to be Weibull distributed. For a mill with medium (short) blades, the relative power level is assumed to be 0.10 (0.15) units higher than each corresponding value for the long blades (but never larger than 1).