MVE165/MMG631
Linear and Integer Optimization with Applications
Lecture 1
Introduction; course map; operations research; modelling; graphic solution

Ann-Brith Strömberg

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Staff

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- **Assignment advisement**
  - Caroline Granfeldt
  - Quanjiang Yu
  - Zuzana Nedělková
- **Guest lecturers**
  - Ola Carlson (Professor. Sustainable Electric Power Production)
  - TBD

Course homepage, PingPong and TimeEdit

- **Course homepage**
  - `www.math.chalmers.se/Math/Grundutb/CTH/mve165/1617`
  - Details, information on assignments and computer exercises, deadlines, lecture notes, problem solving sessions etc
  - Will be updated with new information (at least) every week
- **PingPong**
  - `https://pingpong.chalmers.se`
  - Software download (AMPL & CPLEX/optimization solvers)
  - Course representatives & evaluation
  - All hand-in of assignments
- **TimeEdit**
  - Check TimeEdit continuously for rooms (lectures, eercises, labs)
  - Dependent on # students that attend

Organization

- **Lectures** – mathematical optimization theory
- **Computer exercise** – learn to use software solvers (week 12)
- **Problem solving sessions** – hands-on exercises, two parallel groups (Wed 8–10 OR Thu 10–12; see TimeEdit)
- **Assignments** – modelling, use solvers, analyze solutions, write reports, opposition & oral presentation
  - Each assignment is introduced at a lecture (see course plan)
  - *Assignment work should be done in groups of* ≤ 2 *persons*
  - Define your project groups on the PingPong page of MVE165/MMG631
  - The name of the project group must be: *"FirstName1 Surname1 - FirstName2 Surname2"*
  - Students without PingPong access: *contact me by email*

Computer rooms

- **Computer rooms** are reserved (check TimeEdit for rooms and details)
  - most mondays at 13.15–15.00,
  - most wednesdays at 13.15–17.00, and
  - most fridays at 13.15–15.00
- The computer sessions are NOT mandatory
- The computer rooms are NOT large enough for ALL students (~85) simultaneously, but fairly well spread hours are available
- Teachers present only when indicated in the *Course plan* on the home page

Software

- **A computer exercise on linear optimization and software** is found on the homepage (under *Software*). You are highly recommended to perform this exercise to prepare for the assignment work.
- **AMPL-packages** (time limited) to install on your own computer (linux, mac, windows) is available via PingPong. Read the agreement text! Presented in Lecture 2.
- **Matlab**
- **A java-applet for learning the branch-and-bound algorithm** (on the homepage, under *Software*)

Course evaluation process

- Questionnaires will be sent out to all students registered for the course after the exam week
- The examiner calls the course representatives to the introductory, (recommended) middle, and final meetings
- Course representatives (Chalmers)
  - ARVID BJURKLINT, TKTEM
  - FRIDA ERIKSSON, TKTEM
  - OSKAR HOLMSTEDT, TKTEM
  - STEFANUS IVARSSON BERGENHEM, MPSYS
  - JAKOB LINDQVIST, MPENM
  - One or two volountary GU students?
- Decide on a date for the first meeting – during the break

Literature

- **Main course book**:
  - English version: Optimization (2010)
  - Swedish version: Optimeringslära (2008)

  by J. Lundgren, M. Rönnqvist, and P. Värbrand. Studentlitteratur.

- **Exercise book**:
  - English version: Optimization Exercises (2010)
  - Swedish version: Optimeringslära Övningsbok (2008)

  by M. Henningsson, J. Lundgren, M. Rönnqvist, and P. Värbrand. Studentlitteratur.

- Cremona/Studentlitteratur/Adlibris/...
- Also some **hand-outs** (denoted in the lecture notes)

Examination requirements

- Perform **three project assignments** in groups of two students
  - For Assignment 3 there will be two alternatives
- **Written reports** of three assignments
- **A written opposition** to another group's report of Assignment 2 (individual peer review)
- **An oral presentation** of Assignment 3 (weeks 20–21)
- **Presence** at one full oral presentation session
- *To be able to receive a grade higher than 3 or G, the written reports and opposition as well as the oral presentation must be of high quality. Students aiming at grade 4, 5, or VG must also pass an oral exam (week 22)*

Overview of the lectures and course contents

Mathematical subjects

- Linear optimization models, modelling, theory, solution methods, and sensitivity analysis
- Discrete optimization models, properties, and solution methods
- Optimization problems that can be modelled as flows in networks, theory, and solution methods
- Multi-objective optimization
- Mixes of the above
- Overview of non-linear optimization models, properties, and solution methods

Activities

- Applications of optimization
- Mathematical modelling
- Theory – mathematical properties of models
- Solution techniques – algorithms
- Software solvers

Optimization: "Do something as good as possible"

- **Something:** Which are the decision alternatives?
  - $x_{ij} = \begin{cases} 1 & \text{if customer } j \text{ is visited } \textit{directly} \text{ after customer } i \\ 0 & \text{else} \end{cases}$
  - $y_{ik} = \begin{cases} 1 & \text{if customer } i \text{ is visited by vehicle } k \\ 0 & \text{else} \end{cases}$
- **Possible:** What restrictions are there?
  - Each customer should be visited exactly once
  - Time windows, transport needs and capacity, pick up at one customer – deliver at another, different types of vehicles, ...
- **Good:** What is a relevant optimization criterion?
  - Minimize the total distance / travel time / emissions / waiting time / ...

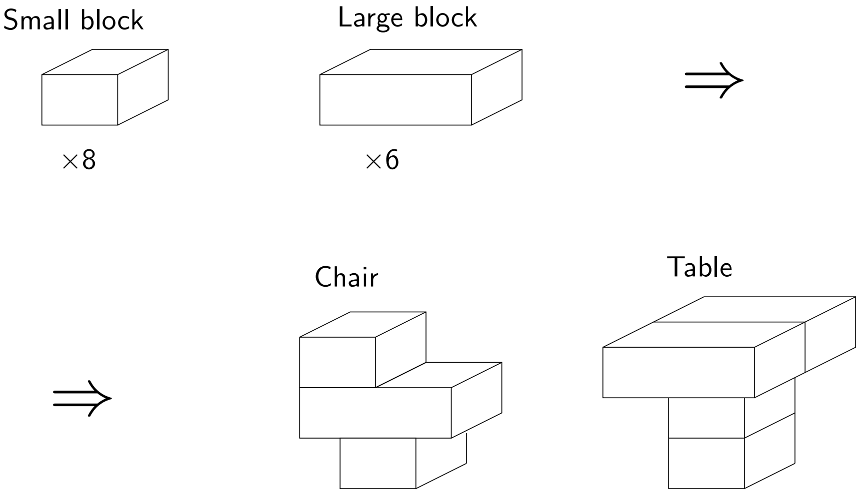
The classical travelling salesperson problem

- Variants of routing problems: e.g., refrigerated goods, transportation service for disabled persons, school buses, hybrid propulsion vehicles (electricity/diesel), ...

Examples of application areas

- **Logistics: production and transport**
  - Optimize routes for transports, snow removal, school buses, ...
  - Location of stores
  - Planning of wood cut and transports
  - Packing of containers
  - Production planning and scheduling
  - Dimensioning of batteries and electric motors in routing applications
- **Energy**
  - Energy production planning
  - Investment in energy production technology
  - Location of power plants and infrastructure
- **Finance**
  - Financial risk management
  - Portfolio optimization
  - Investment planning
- **Medicine**
  - Compute radiation directions/intensities for cancer treatment
  - Reconstruct images from x-ray measurements

A manufacturing example: Produce tables and chairs from two types of blocks



A manufacturing example, continued

- A chair is assembled from one large and two small blocks
- A table is assembled from two blocks of each size
- Only 6 large and 8 small blocks are avaliable
- A table is sold at a revenue of 1600:-
- A chair is sold at a revenue of 1000:-
- Assume that all items produced can be sold and determine an optimal production plan

A mathematical optimization model

- **Something** – What decision alternatives? $\Rightarrow$ Variables

$$\begin{aligned} x_1 &= \text{number of tables produced and sold} \\ x_2 &= \text{number of chairs produced and sold} \end{aligned}$$

- **Possible** – What restrictions? $\Rightarrow$ Constraints
  - Maximum supply of large blocks: 6

$$\boxed{2x_1 + x_2 \leq 6}$$

  - Maximum supply of small blocks: 8

$$\boxed{2x_1 + 2x_2 \leq 8}$$

  - Physical restrictions (also: x_1, x_2 integral)

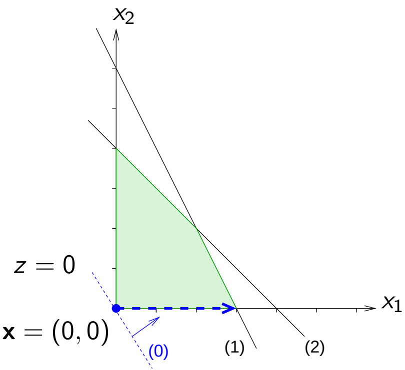
$$\boxed{x_1, x_2 \geq 0}$$

- **Good** – Relevant optimization criterion?$\Rightarrow$Objective function
  - Maximize the total revenue

$$\boxed{1600x_1 + 1000x_2 \to \max}$$

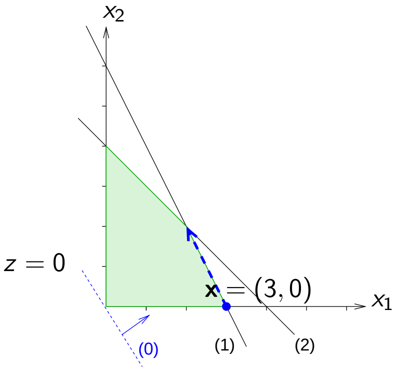
Solve the model using LEGO and marginal values

- Start at no production:
  $x_1 = x_2 = 0$
  Use the "best marginal profit" to choose the item to produce
  - x_1 has the highest marginal profit (1600:-/table)
    $\Rightarrow$ produce as many tables as possible
  - At $x_1 = 3$: no more large blocks left



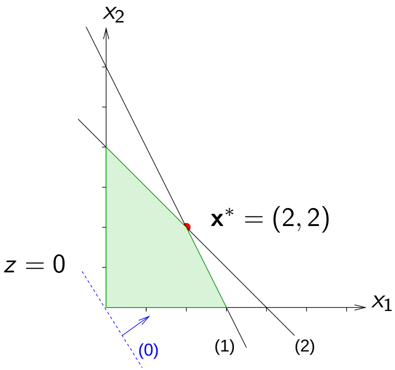
Solve the model using LEGO and marginal values

- The marginal value of x_2 is now 200:- since taking apart one table (-1600:-) yields two chairs ($+2000$:-) $\Rightarrow$ 400:-/2 chairs
  - Increase x_2 maximally $\Rightarrow$ decrease x_1
  - At $x_1 = x_2 = 2$: no more small blocks



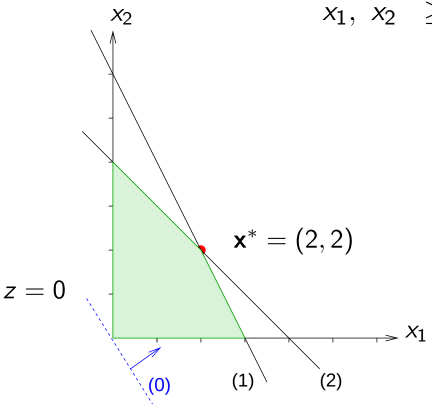
Solve the model using LEGO and marginal values

- The marginal value of x_1 is negative (to build one more table one has to take apart two chairs $\Rightarrow -400$:-)
  The marginal value of x_2 is -600:- (to build one more chair one table must be taken apart)
  $\Longrightarrow$ Optimal solution:
  $x_1 = x_2 = 2$



Geometric solution of the model

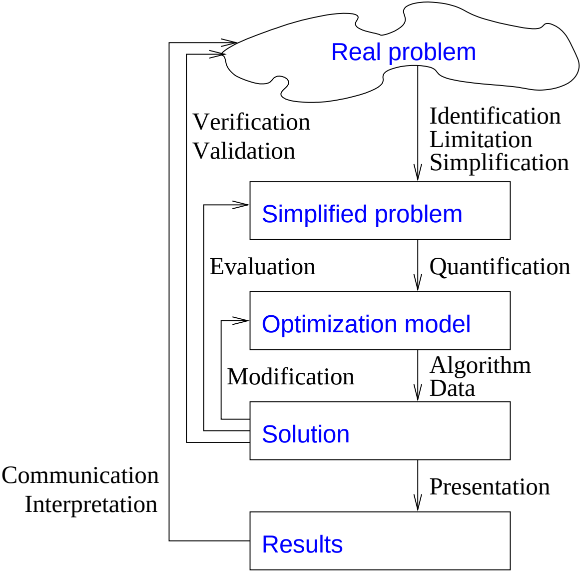
$$
\begin{array}{lcrcrcll}
\text{maximize} & z = & 1600x_1 & + & 1000x_2 & & & (0)\\
\text{subject to} & & 2x_1 & + & x_2 & \leq & 6 & (1)\\
 & & 2x_1 & + & 2x_2 & \leq & 8 & (2)\\
 & & & & x_1,\ x_2 & \geq & 0 &
\end{array}
$$



Operations Research (OR) Swedish: Operationsanalys

- Scientific view on problem solving regarding complex systems
- *"OR means to prepare a foundation for rational decisions by utilizing systematic scientific methods and—when possible and meaningful—by utilizing quantitative models"*
- The problem is considered as a system of components which interact and influence each other
- The activities studied are described by models, used to
  - better understand the depicted system,
  - understand the consequences of different decisions, and
  - choose the "best" alternative due to some criterion.

The process of optimization

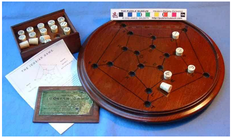


History of Operations Research

- During World War II decision problems became systematically treated: *Operations Research*
- After the war: use of operations research for *civil operations*
- The ideas spread to many countries
- Early operations research include *inventory planning*

A few moments in optimization history

- Euler (1735): Seven bridges of Köningsberg (graph theory)
- Gauss (1777–1855): 1st optimization technique – *steepest descent*
- W.R. Hamilton (1857): "icosian game"
  $\Rightarrow$ *the travelling salesperson problem* (Hamilton cycle)



- L.V. Kantorovich (1939): A linear model for *optimization of plywood manufacturing* and an *algorithm* for its solution
- George B. Dantzig (1947): Linear programming – *the simplex algorithm* (exponential time)
  - Program $\Leftrightarrow$ military training and logistics schedules
- Kantorovich and Koopmans (1975): Nobel Memorial Prize in Economic Sciences
- N. Karmarkar (1984) algorithm for linear programming (polynomial time)

General mathematical optimization models

$$\left[\begin{array}{lll} \text{minimize or maximize} & f(x_1,\ldots,x_n) & \\ \text{subject to} & g_i(x_1,\ldots,x_n) & \left\{\begin{array}{c}\le\\=\\\ge\end{array}\right\} \quad b_i, \quad i=1,\ldots,m \end{array}\right]$$

- $x_1,\ldots,x_n$ are the decision variables ($x_j \in \mathbb{R}$)
- $f:\mathbb{R}^n \mapsto \mathbb{R}$ and $g_1,\ldots,g_m$ ($g_i:\mathbb{R}^n \mapsto \mathbb{R}$) are given functions of the decision variables
- $b_1,\ldots,b_m$ are specified constant parameters ($b_i \in \mathbb{R}$)
- The functions can be nonlinear, e.g. quadratic, exponential, logarithmic, non-analytic, ...
- In general, linear forms are more tractable than non-linear

Linear optimization models (programs)

- The capacity expansion model is a linear program (LP), i.e., all relations are described by linear forms
- A general linear program:

$$\left[\begin{array}{lrcll} \text{min or max} & c_1x_1 + \ldots + c_nx_n & & & \\ \text{subject to} & a_{i1}x_1 + \ldots + a_{in}x_n & \left\{\begin{array}{c}\leq\\=\\\geq\end{array}\right\} & b_i, & i = 1, \ldots, m \\ & x_j & \geq & 0, & j = 1, \ldots, n \end{array}\right]$$

- The non-negativity constraints on x_j, $j = 1, \ldots, n$ are not necessary, but usually assumed (reformulation always possible)

Discrete/integer/binary modelling

- A variable is called *discrete* if it can take only a countable set of values, e.g.,
  - Continuous variable: $x \in [0, 8] \iff 0 \leq x \leq 8$
  - Discrete variable: $x \in \{0, 4.4, 5.2, 8.0\}$
  - *Integer* variable: $x \in \{0, 1, 4, 5, 8\}$
- A *binary* variable can only take the values 0 or 1, i.e., all or nothing

  E.g., a wind-mill can produce electricity only if it is built
  - Let $y = 1$ if the mill is built, otherwise $y = 0$
  - Capacity of a mill: C
  - Production $x \leq Cy$ (also limited by wind force etc.)
- In general, models with only continuous variables are more tractable than models with integrality/discrete requirements on the variables, but exceptions exist! More about this later.