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Mathematical Sciences
Optimization
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MVE165
MMG631
Linear and integer optimization
with applications

Assignment information
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Assignment 1: Biodiesel supply chain

Below is a description of the biodiesel supply chain problem such that the total profit from supplying the demand of biodiesel is maximized. The assignment tasks are to

- formulate a linear optimization model of the problem described,
- solve the problem using AMPL and CPLEX (or some other solver), and
- analyse the results and answer a number of given questions.

Study the Modeling Language for Mathematical Programming AMPL and the solver CPLEX using the following links or the recommended exercise on linear optimization and software from the course homepage before you start solving the exercises.

- `http://www.ampl.com/BOOK/download.html`
- `http://www.ampl.com/BOOKLETS/amplcplex122userguide.pdf`

To pass the assignment you should (in groups of two persons) write a self-contained report on the project work, in which you give satisfactory answers to the questions below, in the form of a PDF file. You should write the report on a computer, preferably using LaTeX. You shall also estimate the number of hours spent on this assignment and note this in your report. You may discuss the problem with other students. However, each group must hand in their own solution. The report will be checked for plagiarism via `http://www.urkund.com`.

The questions 1, 2, and 3a–3f are mandatory. In addition, students aiming at grade 4, 5, or VG must answer the questions 3g–3h, and the quality of the report must be high.

The file containing your report shall be called **Name1-Name2-Ass1.pdf**, where "Name$k$", $k = 1, 2$, is your respective family name. **Do not forget to write the authors' names also inside the report.** The report should be at most 3–4 pages plus illustrating diagrams and it must be

submitted in PingPong at latest Wednesday 18th of April 2018, 23:55.

In addition, each student must hand in an individually written report describing the distribution of the project work within the group and how the cooperation has worked out. This report must be

submitted in PingPong on 2018-04-19 between 06:00 and 23:55.

Problem background

The background of this problem comes from a study of the biofuels supply chain in Greece performed during the years 2006–2010 by Papapostolou et al. and reported in [4]. The numerical data are collected from [1, 3, 2].

The following sections present a short description of biodiesel production. After this follows a short description of the plants that can be used and of the final products with their demands. The description includes prices, yields, availabilities of different sources, demands, and data regarding the different production processes.

1. Biofuels supply chain

The biodiesel is produced in a two-step process, first the extraction of vegetable oil from seeds, after this the transesterification in order to produce the pure biodiesel.

In the extraction phase the vegetable oils is gained from the seeds through a simple screw press. The vegetable oil content in the respective kind of seeds is presented in Table 1.

All extracted vegetable oil is transesterificated to produce biodiesel. The transesterification is a process in which the vegetable oil is being reacted with methanol. The biodiesel is the product resulting from this reaction. The produced biodiesel has to be refined to remove impurities. To produce 0.9 l of biodiesel we need 1 l of vegetable oil and 0.2 l of methanol. The price of methanol is 1.5 €/l.

The refined biodiesel is to be blended with the petrol diesel to different concentrations and sold to the end customers. The petrol diesel has to be purchased for 1 €/l. This price is lower because we do not have to pay tax and we can use the wholesale price. But there is only 150 000 l of petrol diesel available.

2. Crops

First of all, the producer has to cultivate suitable crops on the available area of 1 600 ha. Soya, sunflower, and cotton were selected as the most suitable in this case. Each crop yields an expected amount of seeds, measured in tonnes per ha. The field has to be watered during the cultivation to produce the expected yields. The water demand for each crop is measured in megalitres per ha (Ml/ha). The amount of available water is limited by the upper bound of 5 000 Ml. The relevant data are presented in Table 1.

Crop	Yield [t/ha]	Water demand [Ml/ha]	Oil content [l/kg]
Soybeans	2.5	5.0	0.178
Sunflower seeds	1.5	4.2	0.216
Cotton seeds	0.8	1.0	0.433

Table 1: Crops data

3. Final products

Biodiesel is meant to be used in standard diesel engines. Biodiesel can be used alone, or blended with petrol diesel. Blends of biodiesel are products most commonly distributed for use in the retail fuel marketplace. These blends are denoted by its "B" factor to state the amount of biodiesel in any fuel mix.

We focus on three specific blends in this problem B5, B30, and B100 with the compositions and prices listed in Table 2. The first most common blend B5 is in fact the usual diesel sold in EU. There is an EU regulation that the minimum percentage of biodiesel in diesel should be 5%. This blend can be used in all cars and trucks with diesel engines currently on the market with no effect on the engine manufacturers' warranty. Peugeot and Citroën guarantee that also the blend B30 can be used in their cars, and Scania and Volkswagen can even handle B100 in their cars and trucks. Different European countries use different tax policies, but usually a higher amount of biodiesel in the blend means lower tax, the specific percentages to be used in this case study are listed in Table 2.

Product	Biodiesel [%]	Price [€/l]	Tax [%]
B5	5	1.42	20
B30	30	1.27	5
B100	100	1.15	0

Table 2: Products data

The biodiesel producer has to meet the overall demand of 280 000 l of fuels to be delivered to the end customers.

Exercises to perform and questions to answer

1. Formulate a linear programming model that seeks to maximize the profit from biodiesel production in order to fulfill all the described constraints and demands. Include an illustration of the product flows of the problem, the variables must be clearly defined and connected to the illustration. The objective function and the constraints should be clearly described. Use a general notation for the values of coefficients, such as p_i for the proportion of biodiesel in product i.

2. Implement the model developed in 1 in AMPL and solve it using CPLEX. Present your results and findings, in particular state the optimal solution with its optimal objective value and discuss if the obtained solution is reasonable. The AMPL implementation should consist of the model file, the data file, and the run file. The run file should contain the following two directives before the solve directive, in order to preserve the information needed for the sensitivity analysis in 3.

   ```
   option cplex_options 'sensitivity';
   option presolve 0;
   ```

3. Perform a sensitivity analysis of your model, and in particular answer the following questions. In each of these questions the variation of data and/or constraints shall be made starting from the original model and data, as implemented in 2. For each of the questions, the resulting new solutions should be compared with the original solution (from 2); mathematical analyses, discussions, and reflections on the outcomes should be presented.

   (a) Find out how much the petrol diesel availability, water availability, and area availability can be reduced while staying feasible. Vary one quantity at a time and explain your findings. Then, try to find out how the corresponding three limits are related.

   (b) How much would we gain from marginal improvements of the limiting availabilities (petrol diesel, water, area)? Consider one limit at a time. Answer this question by making small increments (one at a time) of the limiting availabilities and resolve the model to see how the total profit changes. Compare the results with the values of the dual variables for the limiting constraints in the optimal solution to original problem.

   (c) What and how much has to be changed in the data of the problem to make the planting of sunflower be non-zero in the optimal solution?

   (d) Assume that the price of petrol diesel is increased to 1.20 €/l. How does the total profit change? What is then the optimal solution?

   (e) Suppose that the tax-policy changes. Vary the tax percentages and find out at which values of these there are "significant" changes in the solution, and describe these changes. How does the objective value change with the tax levels?

(f) Due to the uncertainty of the weather, the water demand of the crops may vary. Let the water demand for all the crops vary similarly at the same time since they are all exposed to the same weather. How does the optimal solution change? How much is the water demand for soybeans allowed to increase before the optimal solution changes?

(g) The biodiesel has to have a guaranteed cetane number indicating its quality—the relation between the cetane number and the quality of biodiesel is studied in, e.g., [5]. Describe how the required cetane number for each product can be incorporated into your model and how the distribution over the resulting products can be controlled (do not implement in AMPL).

(h) Discuss how a more "sustainable" (or, environmental friendly) objective function can be defined, what additional data would be needed and what could be the optimal solution for such an objective.

References

[1] C. Charles, I. Gerasimchuk, R. Bridle, T. Moerenhout, E. Asmelash, and T. Laan. Biofuels–At What Cost? A review of costs and benefits of EU biofuel policies. Technical report, International Institute for Sustainable Development, 2013. Available as `http://www.iisd.org/gsi/sites/default/files/biofuels_subsidies_eu_review.pdf`.

[2] J. Doorenbos, A. H. Kassam, and C. I. M. Bentvelsen. *Yield response to water*. FAO irrigation and drainage paper. Food and Agriculture Organization of the United Nations, 1979.

[3] H. E. Gridley and J. B. Smithson. Oil content of cotton seed in Northern Nigeria. *The Journal of Agricultural Science*, 88:731–736, 1977.

[4] C. Papapostolou, E. Kondili, and J. K. Kaldellis. Development and implementation of an optimisation model for biofuels supply chain. *Energy*, 36(10):6019–6026, 2011.

[5] J. Van Gerpen. Cetane number testing of biodiesel. In *Proceedings, Third Liquid Fuel Conference: Liquid Fuel and Industrial Products from Renewable Resources*, pages 197–206, 1996.