

MVE165/MMG631

Linear and integer optimization with applications

Lecture 8b

Maintenance scheduling optimization;
Assignment 2

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Maintenance optimization — a background

- Invitation 2000 from Volvo Aero Corporation (VAC, nowadays GKN Aerospace): maintenance of the RM12 jet engine
- Paired PhD project between applied math/optimization and math statistics/material fatigue and reliability
- Optimization PhD student: a model for opportunistic maintenance; superior to simpler policies
- Mathematical statistics student: models for determining life distributions based on crack growth
- Continuation projects: GKN; planning maintenance of components in wind power plants and scheduling of rail grinding

Maintenance in industry

- *Optimal maintenance* = obtain reliability at the least cost
- *Maintenance* costs/year:
14000 billion SEK (in EU), 275 billion SEK (in Sweden)
- *Maintenance* is often seen merely as a cost
- *Maintenance* is sometimes done too often—inspections and measurements may damage the systems
- Sometimes—like with road/rail infrastructure and “Miljonprogramhusen”—it is performed seldom
- Truth: well performed *maintenance* is an investment in availability and safety



Maintenance principles

- **Preventive maintenance (PM)**: actions that prevent failure
- **Corrective maintenance (CM)**: actions after failure, repairs
- **Condition based maintenance (CBM)**: measurements → predictions → actions according to a maintenance principle
- **Opportunistic maintenance (OM)**: when maintenance must be performed, make also some (additional) preventive maintenance actions

A simple example, I

A system with n components

- Life of component i : T_i time units (intervals)
- Time horizon: T time units (e.g. contract period)
- Cost of a spare component of type i at time t : c_{it} monetary units
- Cost for performing any maintenance at time t : d_t monetary units

A simple example, II

Variables are logical – do something or not

The model uses binary variables:

$$x_t = \begin{cases} 1, & \text{if “something” is performed at time } t \\ 0, & \text{otherwise} \end{cases}$$

A decision often implies other necessary decisions

- Example: if component i shall be replaced at time t maintenance must be performed
- Such logical relations are equivalent to linear constraints:

$$\text{if A then B} \quad \iff \quad x_A \leq x_B$$

The basic replacement problem, I

Goal: minimize the total cost for keeping the system working during the contract period:

Mathematical model

$$\underset{(x,z)}{\text{minimize}} \quad \sum_{t=1}^T \left(\sum_{i=1}^N c_{it} x_{it} + d_t z_t \right), \quad (1a)$$

$$\text{subject to} \quad \sum_{t=\ell+1}^{\ell+T_i} x_{it} \geq 1, \quad \ell = 0, \dots, T - T_i, \quad i = 1, \dots, N, \quad (1b)$$

$$x_{it} \leq z_t, \quad t = 1, \dots, T, \quad i = 1, \dots, N, \quad (1c)$$

$$x_{it} \geq 0, \quad t = 1, \dots, T, \quad i = 1, \dots, N, \quad (1d)$$

$$z_t \leq 1, \quad t = 1, \dots, T, \quad (1e)$$

$$x_{it}, z_t \in \{0, 1\}, \quad t = 1, \dots, T, \quad i = 1, \dots, N \quad (1f)$$

The basic replacement problem, II

Objective (1a)

Minimize the total cost of having a working system during the contract period

Constraint (1b)

For any given item i in the system, the component must be replaced at some point during every time interval of T_i time steps

Constraint (1c)

No replacement can be performed at time t without paying the fixed cost d_t (for a maint. operation); once we pay, any maint. action becomes possible (at no extra fixed cost) at time t

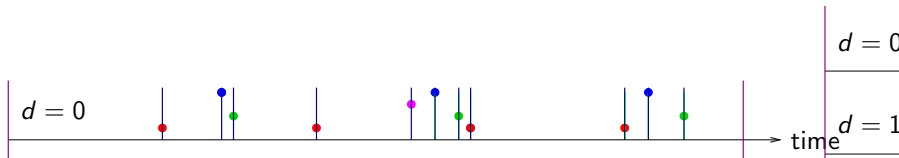
Constraints (1d)–(1f)

Ensure that the variables take only meaningful values

Opportunistic maintenance or not?

Example: four components with different prices and lives

- A *replacement* is marked with a dot; its colour represents the type of component replaced
- The larger the fixed cost, the more beneficial *opportunistic* maintenance becomes; also more items are replaced



Constraint structure—example

Time horizon: $T = 8$. Component #3: $T_3 = 4$

$$\sum_{t=l+1}^{l+T_3} x_{3t} \geq 1, \quad l = 0, \dots, T - T_3$$

$$\iff \sum_{t=l+1}^{l+4} x_{3t} \geq 1, \quad l = 0, \dots, 4$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_{31} \\ x_{32} \\ \vdots \\ x_{38} \end{bmatrix} \geq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Property I: the replacement problem is NP-hard

Theorem

Set covering is polynomially reducible to the replacement problem

- This essentially means that we *cannot* expect to find an optimal solution in a time that is proportional to a polynomial function of the problem size (i.e., $T(N + 1)$ variables and $\approx 4NT$ constraints)
- Basic complexity theory: Chapter 2.6 in the course book

Property II: with fixed z the problem over x is easy

- The constraint matrix of (1b), (1d) (concerning only the variables x) has the “consecutive ones” property
- ⇒ For fixed values of z , the problem over x can be solved as a *linear program*
- For each i , the linear programming dual problem can be solved by a “greedy” algorithm ⇒ primal solution by complementarity; see [a], Algorithm 1, page 297
- The latter is typically 5–40 times faster than solving as a general linear program, and 25–400 times faster when costs are monotone with time (i.e., $\forall t$ either $c_{it} \leq c_{i,t+1}$ or $c_{it} \geq c_{i,t+1}$); see [a], Algorithm 2, page 299

[a] T. Almgren, N. Andréasson, M. Patriksson, A.-B. Strömberg, A. Wojciechowski, M. Önnheim (2012): *The opportunistic replacement problem: theoretical analyses and numerical tests*, Mathematical Methods of Operations Research, 76(3) pp. 289–319.

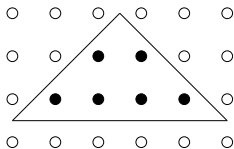
Property III: all inequalities define facets*

An ideal formulation: the convex hull of all feasible integer points

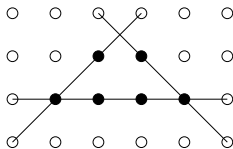
$$X = \{ \mathbf{x}^1, \dots, \mathbf{x}^P \}, \text{ where } \mathbf{x}^p \text{ are integer vectors}$$

$$X^{\text{conv}} = \left\{ \sum_{p=1}^P \mathbf{x}^p \lambda_p \mid \sum_{p=1}^P \lambda_p = 1; \lambda_p \geq 0, p = 1, \dots, P \right\}$$

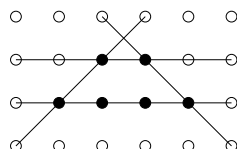
No inequalities
define facets



All inequalities
define facets



An integral
polyhedron (ideal)



* See [a], Section 5.1–5.2

A generalized model

New variable definition

Define the set

$$\mathcal{I} := \{ (s, t) \mid 0 \leq s < t \leq T + 1; s, t \in \mathbb{Z} \}$$

of *replacement intervals* and introduce the variables

$$x_{st}^i = \begin{cases} 1, & \text{if component } i \text{ receives PM at the} \\ & \text{times } s \text{ and } t, \text{ and not in-between,} \\ 0, & \text{otherwise,} \end{cases} \quad \begin{array}{l} i \in \mathcal{N}, \\ (s, t) \in \mathcal{I}, \end{array}$$

and

$$z_t = \begin{cases} 1, & \text{if maintenance occurs at time } t, \\ 0, & \text{otherwise,} \end{cases} \quad t \in \mathcal{T}.$$

A generalized model

$$\text{minimize} \quad \sum_{t \in \mathcal{T}} d_t z_t + \sum_{i \in \mathcal{N}} \sum_{(s,t) \in \mathcal{I}} c_{st}^i x_{st}^i, \quad (2a)$$

$$\text{subject to} \quad \sum_{s=0}^{t-1} x_{st}^i \leq z_t, \quad i \in \mathcal{N}, t \in \mathcal{T}, \quad (2b)$$

$$\sum_{s=0}^{t-1} x_{st}^i = \sum_{r=t+1}^{T+1} x_{tr}^i, \quad i \in \mathcal{N}, t \in \mathcal{T}, \quad (2c)$$

$$\sum_{t=1}^{T+1} x_{0t}^i = 1, \quad i \in \mathcal{N}, \quad (2d)$$

$$x_{st}^i \in \{0, 1\}, \quad i \in \mathcal{N}, (s, t) \in \mathcal{I}, \quad (2e)$$

$$z_t \in \{0, 1\}, \quad t \in \mathcal{T}. \quad (2f)$$

On the GKN project

Aircraft engines are expensive

- Spare components cost up to 2 MSEK
 - Total cost of maintenance of one engine: 15–30 MSEK
 - Maximizing “time on wing” is important, both for civil and military aircraft
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- The aircraft engine RM12 consists of 7 modules and 61 components in total
 - A mathematical model has been constructed for the entire engine maintenance, including work costs for (dis)assembling the necessary modules and components for each maintenance occasion
 - This model has slightly less than 6000 binary variables

Results on the GKN problems

An individual engine module with 10 components

- Cost reduction: 35%
- Reduction of # maintenance occasions: 7%

as compared with a simple policy similar to that used at GKN

A complete engine with 7 modules (61 components)

- Cost reduction compared to maintaining (optimally) each individual module: 12%
- Reduction of # maintenance occasions: 60%

Product development

Found 5 components which can potentially reduce maintenance costs more than 5% through prolonged lives

Assignment 2: Maintenance Scheduling

Assignment tasks in summary

- Study and compare the two ILP models for maintenance planning
- Elaborate with the models - w.r.t. facets, integrality property, time horizon
- Heuristic solutions—local search method
- Add side constraints—modelling additional properties
- Students aiming at grade 4, 5, or VG must answer ALL the questions
- *Deadline* for handing in report: *May 3 at 9:30*
- *Hand in* the same report, individually: *May 3, 12:00–17:00*
- Written opposition (peer review, individual) on another report.
Deadline: May 7