Chalmers University of Technology	MVE165
University of Gothenburg	MMG631
Mathematical Sciences	Linear and integer optimization
Optimization	with applications
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Assignment 1: Biodiesel supply chain

The questions 1, 2, and 3a–3f are mandatory. Students aiming at grade 4, 5, or VG must also answer the questions 3g–3i, and the quality of the assignment report must be high.

Problem background

The background of this problem comes from a study of the biofuels supply chain in Greece performed during the years 2006–2010 by Papapostolou et al. and reported in [4]. The numerical data are collected from [1, 3, 2], then slightly modified.

The following sections present a short description of biodiesel production. After this follows a short description of the plants that can be used and of the final products with their demands. The description includes prices, yields, availabilities of different sources, demands, and data regarding the different production processes.

1. Biofuels supply chain

The biodiesel is produced in a two-step process, first the extraction of vegetable oil from seeds, after this the transesterification in order to produce the pure biodiesel.

In the extraction phase the vegetable oils is gained from the seeds through a simple screw press. The vegetable oil content in the respective kind of seeds is presented in Table 1.

All extracted vegetable oil is transesterificated to produce biodiesel. The transesterification is a process in which the vegetable oil is being reacted with methanol. The biodiesel is the product resulting from this reaction. The produced biodiesel has to be refined to remove impurities. To produce 0.9 l of biodiesel we need 1 l of vegetable oil and 0.2 l of methanol. The price of methanol is 1.5 €/l.

The refined biodiesel is to be blended with the petrol diesel to different concentrations and sold to the end customers. The petrol diesel has to be purchased for 1 €/l. This price is lower because we do not have to pay tax and we can use the wholesale price. But there is only 150 000 l of petrol diesel available.

2. Crops

First of all, the producer has to cultivate suitable crops on the available area of 1 600 ha. Soya, sunflower, and cotton were selected as the most suitable in this case. Each crop yields an expected amount of seeds, measured in tonnes per ha. The field has to be watered during the cultivation to produce the expected yields. The water demand for each crop is measured in megalitres per ha (Ml/ha). The amount of available water is limited by the upper bound of 5 000 Ml. The relevant data are presented in Table 1.

Crop	Yield [t/ha]	Water demand [Ml/ha]	Oil content [l/kg]
Soybeans	2.6	5.0	0.178
Sunflower seeds	1.4	4.2	0.216
Cotton seeds	0.9	1.0	0.433

Table 1: Crops data

3. Final products

Biodiesel is meant to be used in standard diesel engines. Biodiesel can be used alone, or blended with petrol diesel. Blends of biodiesel are products most commonly distributed for use in the retail fuel marketplace. These blends are denoted by its "B" factor to state the amount of biodiesel in any fuel mix.

We focus on three specific blends in this problem B5, B30, and B100 with the compositions and prices listed in Table 2. The first most common blend B5 is in fact the usual diesel sold in EU. There is an EU regulation that the minimum percentage of biodiesel in diesel should be 5%. This blend can be used in all cars and trucks with diesel engines currently on the market with no effect on the engine manufacturers' warranty. Peugeot and Citroën guarantee that also the blend B30 can be used in their cars, and Scania and Volkswagen can even handle B100 in their cars and trucks. Different European countries use different tax policies, but usually a higher amount of biodiesel in the blend means lower tax, the specific percentages to be used in this case study are listed in Table 2.

The biodiesel producer has to meet the overall demand of 280 000 l of fuels to be delivered to the end customers.

Product	Biodiesel [%]	Price [€/l]	Tax [%]
B5	5	1.43	20
B30	30	1.29	5
B100	100	1.16	0

Table 2: Products data

Exercises to perform and questions to answer

1. Formulate a linear programming model that seeks to maximize the profit from biodiesel production in order to fulfill all the described constraints and demands. Include an illustration of the product flows of the problem, the variables must be clearly defined and connected to the illustration. The objective function and the constraints should be clearly described. Use a general notation for the values of coefficients, such as p_i for the proportion of biodiesel in product i.

2. Implement the model developed in 1 in Julia/JuMP and solve it using Gurobi. Present your results and findings, in particular state the optimal solution with its optimal objective value and discuss if the obtained solution is reasonable. The implementation should consist of a model file, a data file, and a run file.

3. Perform a sensitivity analysis of your model, and in particular answer the following questions. In each of these questions the variation of data and/or constraints shall be made starting from the original model and data, as implemented in 2. For each of the questions, the resulting new solutions should be compared with the original solution (from 2); mathematical analyses, discussions, and reflections on the outcomes should be presented.

   (a) Find out how much the petrol diesel availability, water availability, and area availability can be reduced while staying feasible. Vary one quantity at a time and explain your findings. Then, try to find out how the corresponding three limits are related.

   (b) How much would we gain from marginal increments of the limiting availabilities (petrol diesel, water, area)? Consider one limit at a time. Answer this question by making small increments (one at a time) of the limiting availabilities and resolve the model to see how the total profit changes. Compare the results with the values of the dual variables for the limiting constraints in the optimal solution to original problem.

   (c) What and how much has to be changed in the data of the problem to make the planting of sunflower be non-zero in the optimal solution?

   (d) Assume that the price of petrol diesel is increased to 1.20 €/l. How does the total profit change? What is then the optimal solution?

(e) Suppose that the tax-policy changes. Vary the tax percentages and find out at which values of these there are "significant" changes in the solution, and describe these changes. How does the objective value change with the tax levels?

(f) Due to the uncertainty of the weather, the water demand of the crops may vary. Let the water demand for all the crops vary similarly at the same time since they are all exposed to the same weather. How does the optimal solution change? How much is the water demand for soybeans allowed to increase before the optimal solution changes?

(g) The optimal value of the objective function depends on the values of the right-hand sides of the constraints. Provide a formula for how the optimal objective value changes when making a small increment (or reduction) of one of the limiting availabilities (i.e., one of the right-hand sides). Furthermore, what is a necessary condition for the basis to remain optimal when the value of the right-hand side is changed? Provide a formula describing this condition.

Then, consider again question 3b. What is the shadow price for the amount of available water? What happens with the shadow prices when you change the amount of available water to 1000 Ml? Illustrate the mathematical properties of the objective as a function of variations in the right-hand side. How can the shape of the function be described in a mathematical sense?

(h) The biodiesel has to have a guaranteed cetane number indicating its quality—the relation between the cetane number and the quality of biodiesel is studied in, e.g., [5]. Describe how the required cetane number for each product can be incorporated into your model and how the distribution over the resulting products can be controlled (do not implement in JuMP).

(i) Discuss how a more "sustainable" (or, environmental friendly) objective function can be defined, what additional data would be needed and what could be the optimal solution for such an objective.

References

[1] C. Charles, I. Gerasimchuk, R. Bridle, T. Moerenhout, E. Asmelash, and T. Laan. Biofuels–At What Cost? A review of costs and benefits of EU biofuel policies. Technical report, International Institute for Sustainable Development, 2013. Available as `http://www.iisd.org/gsi/sites/default/files/biofuels_subsidies_eu_review.pdf`.

[2] J. Doorenbos, A. H. Kassam, and C. I. M. Bentvelsen. *Yield response to water*. FAO irrigation and drainage paper. Food and Agriculture Organization of the United Nations, 1979.

[3] H. E. Gridley and J. B. Smithson. Oil content of cotton seed in Northern Nigeria. *The Journal of Agricultural Science*, 88:731–736, 1977.

[4] C. Papapostolou, E. Kondili, and J. K. Kaldellis. Development and implementation of an optimisation model for biofuels supply chain. *Energy*, 36(10):6019–6026, 2011.

[5] J. Van Gerpen. Cetane number testing of biodiesel. In *Proceedings, Third Liquid Fuel Conference: Liquid Fuel and Industrial Products from Renewable Resources*, pages 197–206, 1996.