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Mathematical Sciences	Linear and integer optimization
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Assignment 3b: The Scandinavian electricity system

The problem addressed in the assignment is to design a new Scandinavian electricity system such that the electricity demand is met at all points in time, while minimizing certain objectives.

1 Problem background

In this assignment, a new electricity system in Scandinavia, e.g. Sweden, Norway and Denmark, should be designed. An electricity system is composed of several different generation technologies. Each country defines a region, and it is assumed that the current hydropower capacity installment in the regions is fixed. New capacity investments are, however, possible in wind, nuclear, and fossil fuel (i.e. coal and natural gas) power plants. Moreover, each region only considers aggregated continuous capacity for each of the different generation technologies. This means that, for each region and technology type, all power plants are combined. Thus, you may model this as one giant power plant per technology type in each region.

Note that the concept *capacity* here denotes the maximum possible electricity output during an instant of time. If the capacity is, for example, 5 MW, the maximum electricity output during the time period τ hours is 5τ MWh.

Investments should be minimized but sufficient to cover the electricity demand; see Table 1. A normalized profile, in which each data value (measured in h^{-1}) represents a share of the total demand in a region at a specific time interval during the year, is found in the file `elGen_demand8760.dat` (1 h resolution).

The regions, as noted in the table, have their own demands, but also their own electricity production. Overproduction is allowed. It is, however, possible to freely trade between the regions, although with a transmission loss of 10% of the traded electricity. Trade is also limited by the cable capacity 2200 MW on each transmission line. The system is isolated, which means that no external trade is allowed. Figure 1 illustrates the Scandinavian transmission system.

To be able to generate electricity, proper investments in different power plants are required to take place beforehand. However, when making an investment only its *present value* is known. Thus, to account for interest rates and the

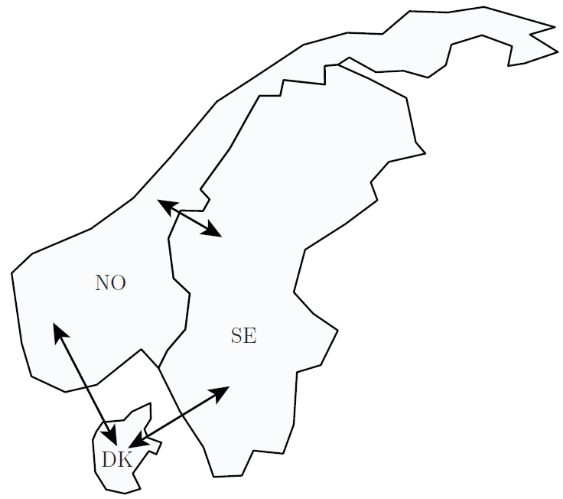

Figure 1: Illustration of the transmission possibilities

Region r	Annual demand [TWh_{el}] d_r
SE	147.2
NO	131.8
DK	35.5

Table 1: The annual electricity demand for each region

life spans of the respective technologies, the annuity payment factor a_n of each investment must be used; it is defined as

$$a_n = \frac{R}{1 - (1 + R)^{-n}}, \tag{1}$$

where $R \in (0, 1]$ denotes interest rate and n [years] the investment's life span.[1]

Some additional necessary properties for the different technologies and their respective fuel types are listed in Tables 2 and 3. Each technology requires a unique fuel type (except for hydro- and wind power, which are fuelless), for which the transformation into electricity has a specific efficiency. Various operation and maintenance (O&M) costs are also added, depending on electricity generation scheme.

Technology specific attributes are given in the following subsections.

[1] The formula (1) is the result of a geometric series

Energy type	Fuel	Efficiency	Life span	Investment costs	O&M costs	Emissions
p	p	η_p	n_p	c_p^{inv}	c_p^{om}	e_p
		[1]	[years]	[k€/MW]	[€/MWh$_{\text{el}}$]	[gCO_2/kWh$_{\text{el}}$]
Nuclear (L)	uranium	0.33	60	61	9.8	115
Nuclear (S)	uranium	0.33	60	61	9.8	115
Hydro	–	1	–	–	1	0
Wind	–	1	25	17	7.5	0
Coal	coal	0.39	50	16	3.6	1000
Gas	natural gas	0.40	20	4.7	2.3	350

Table 2: Plant properties

Fuel	Costs [€/MWh$_{\text{fuel}}$]
p	c_p^{fuel}
uranium	4.8
coal	6.5
natural gas	19.7

Table 3: Fuel properties

1.1 Hydropower

The current installed capacity of hydropower in the Scandinavian electricity system is found in Sweden and Norway, while Denmark has none. The inflow to the reservoirs depends on weather and climatic properties. A normalized profile of this inflow (measured in h^{-1}), as share of the total yearly inflow into reservoirs at a specific time interval, is found in the file `elGen_hydro8760.dat` (1 h resolution). Electricity is then produced by leading the water through turbines; the power extracted depends on the water volume and the height difference between the water's in- and outflow, as illustrated in Figure 2.

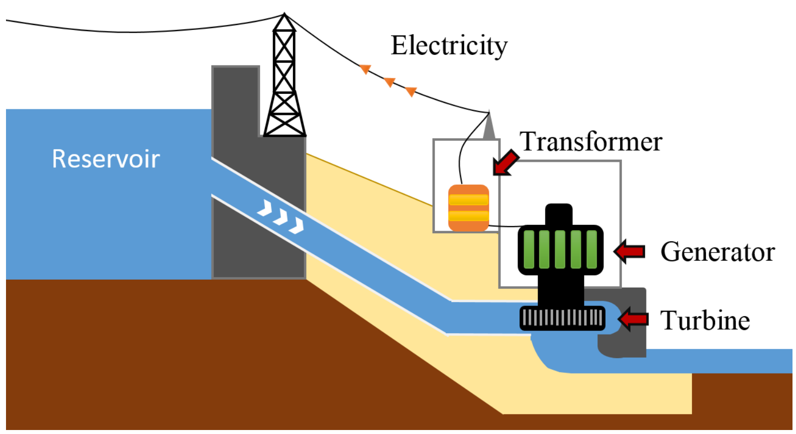

Figure 2: Hydropower electricity generation process

The reservoir level is measured in $\mathrm{MWh_{el}}$. Moreover, the level for the hydro reservoirs when the planning period starts can be any (feasible) value, but it is required to be the same level at the end of the planning period. If the reservoir becomes full, excess water is released through hatches.

Relevant data for the different regions can be found in Table 4.

Region	Installed capacity	Total yearly inflow	Maximum reservoir level
r	h_r^{cap}	h_r^{tot}	H_r
	[GW]	[TWh]	[TWh]
SE	16.2	44	21
NO	30.6	89	55
DK	-	-	-

Table 4: Hydropower properties

1.2 Wind power

Wind power output is intermittent and has a significant variation over shorter time horizons. A wind profile is found in the file `elGen_wind8760.dat` (1 h resolution), unitless and measured as share of installed capacity, which works as an upper limit for the electricity generation.

Not all areas are suitable for installment of wind farms since wind speed and terrain varies across the regions. Thus, for reasons regarding land exploitation, there is an upper limit of the wind power capacity that is available for installment in each region; see Table 5.

Region	Upper capacity limit of wind power [GW]
r	q_r
SE	7
NO	3
DK	10

Table 5: Wind power properties

1.3 Nuclear power

It is possible to invest into nuclear power in the system. The two reactor types small and large can be used, with identical properties besides the capacity, which is shown in Table 6. Several reactors of each type can be bought.

Reactor type	Capacity [MW]
p	m_p
Small	500
Large	850

Table 6: Nuclear power properties

Some physical complications are associated with large short-term variations in output from nuclear power, due to the complexity of turning reactors on and off; this is also very costly. Therefore, to account for these limitations, we assume that the aggregated amount of nuclear power produced in each region and time step must not be less than 80% of the installed capacity.

Exercises to perform and questions to answer

1. Formulate a mixed integer linear programming network flow model that seeks to minimize investment and running costs for electricity production in the Scandinavian electricity system, provided that demand must be met at all times. The model should cover an entire year, which can be assumed to consist of 8760 hours. The model should work regardless of the chosen length of the time step (although in integer multiples of hours). Assume at this point that there are no penalties for emissions, and that the interest rate for investments is 5%.

   Data files containing the profiles for hydropower, wind power and demand are found on the course homepage.

   (*Hint*: performing dimension analysis on each constraint should make it easier to eliminate modeling errors.)

2. Implement the model from 1. in Julia/JuMP. You may use the file `elgen_dat.jl`, found on the course homepage, as a starting point. Solve the model, using Gurobi, for the following cases:

   (a) The time step length is 1 hour.

   (b) The time step length is 3 hours.
   For this case, the profiles (demand, hydro, and wind) should *not* be *averaged* over 3-hour intervals, but instead *sampled* every third hour. This in order to capture the variations in the profile data.
   Hint: use the command `'TIME = 1:3:8760'` to create the set of time steps.

   Present your result and findings—including graphical illustrations of the total annual electricity generation for the different technology types—

and discuss and motivate the differences between the two cases. Especially comment on the CPU time and the number of variables and constraints needed to solve these instances.

3. Discuss, according to the two questions below, how to improve the model from 1. to give a more realistic result. These improvements need not be implemented, but mathematical descriptions of the new parameters, variables and constraints should be included. **Note that the model should still be mixed integer linear!**

   (a) Assume that the aggregated nuclear power in each region does not have to be above 80% of installed capacity at all times. Instead, add the possibility of having everything simultaneously shut down. In other words, the generation from nuclear power in each region should at all times be either 0 or above 80% of installed capacity.

   (b) Assume that there is a limit on how fast the generation can change over time in the coal-, gas- and nuclear power plants. Formulate this constraint for the case when the limit is defined over each consecutive time step.

4. Adjust your mathematical model from 1. to include an upper limit, $e^{\max}$, on the total annual emissions. Reasonable values for this limit are $30 \cdot 10^6 \leq e^{\max} \leq 50 \cdot 10^6$ tonnes CO_2.

5. Implement the model from 4. in Julia/JuMP and solve it using Gurobi. Assume a 3 hour time step. Present your result and findings. Report how the solutions times change and discuss briefly the reason for this.

   To reduce solution times, you can put an upper limit on the elapsed time. This is done by including a parameter when choosing the Gurobi solver in the run-file:

   ```
   setsolver(m, GurobiSolver(TimeLimit = r))
   ```

   where `r` is chosen appropriately.

6. Assuming a time step of 3 hours, consider the multi-objective optimization problem to minimize total costs and minimize emissions. Then, the corresponding solutions from exercises 2(b) and 5. define points on the corresponding Pareto front. If `'TimeLimit = r'` is applied, the solution points correspond to approximate Pareto optimal solutions (which may, on occasion, be Pareto optimal solutions).

   As exercise 5. showed, adding constraints for the emissions to the model increases the solution times severely. Therefore use the *weighted sums method* (simultaneously minimize system costs and emissions under varying weights) to get Pareto points. Construct a graph showing a number of (fairly spread) points on the Pareto front. For ease of interpretation, the costs should be presented in M€ and the emissions in tonnes CO_2. Note that since the model is mixed-integer, the front may be discontinuous.