MVE165/MMG631
Linear and Integer Optimization with Applications
Lecture 1
Introduction; course map; optimization; modelling; graphic solution

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Staff

- **Examiner and lecturer**
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Course homepage, PingPong and TimeEdit

- **Course homepage**
  - `www.math.chalmers.se/Math/Grundutb/CTH/mve165/1819`
  - Details, information on assignments and computer exercises, deadlines, lecture notes, problem solving sessions etc
  - Continuously updated with new information
- **Learning platform: PingPong**
  - `https://pingpong.chalmers.se`
  - All hand-in of assignments
  - Discussion forum about software etc.
  - Course representatives & evaluation
- **TimeEdit**
  - Check TimeEdit continuously for rooms (lectures, problem solving sessions, lab rooms)

Organization

- **Lectures** – mathematical optimization theory; introduction to software and assignments
- **Problem solving sessions** – hands-on exercises, two parallel groups (Wed 8–10 OR Thu 10–12; see TimeEdit)
- **Assignments** – modelling, use software solvers, analyze solutions, write reports, opposition & oral presentation
  - *Assignment work should be done in groups of 2 persons*
  - Define your project groups on the PingPong event for MVE165/MMG631
  - The name of the project group must be: *"FirstName1 Surname1 - FirstName2 Surname2"*
  - Students without PingPong access: *contact me by email*

Computer exercise, lab rooms and software

- **A computer exercise** on software solvers and linear optimization:
  `http://www.math.chalmers.se/Math/Grundutb/CTH/mve165/1819/#Software`
  Perform this exercise during this first study week, in order to prepare for the assignment work
- **Computer rooms** are reserved (check TimeEdit for details)
  - most Mondays at 13.15–15.00,
  - most Wednesdays at 13.15–17.00, and
  - this Friday at 13.15–15.00
- The computer sessions are NOT mandatory
- Teachers will be present

Course evaluation process

- Questionnaires will be sent out to all students registered for the course after the exam week
- The examiner calls the course representatives to the introductory, (recommended) middle, and final meetings
- Course representatives (Chalmers)
  - TBA (in PingPong; the chosen ones will be notified)
  - A volountary GU student?

Literature

- **Main course book**:
  - English version: Optimization (2010)
  - Swedish version: Optimeringslära (2008)

  by J. Lundgren, M. Rönnqvist, and P. Värbrand. Studentlitteratur

- **Exercise book**:
  - English version: Optimization Exercises (2010)
  - Swedish version: Optimeringslära Övningsbok (2008)

  by M. Henningsson, J. Lundgren, M. Rönnqvist, and P. Värbrand. Studentlitteratur

- STORE by Chalmers/Studentlitteratur/Adlibris/...

- Also some **hand-outs** (as indicated in the lecture notes)

Examination requirements

- Perform **three project assignments** in groups of two students
    - For Assignment 3 there will be two alternatives
- **Written reports** of three assignments
- **For each assignment** hand in, individually, a written report on the distribution of the the project work within the group and on how the cooperation has worked out
- **A written opposition** to another group's report of Assignment 2 (individual peer review)
- **An oral presentation** of Assignment 3 (week 22)
- **Presence** at one full oral presentation session
- *To be able to receive a grade higher than 3 or G, the written reports and opposition as well as the oral presentation must be of high quality (mark 2 in PingPong). Students aiming at grade 4, 5, or VG must also pass an oral exam (week 23 or 24)*

Overview of the lectures and course contents

Mathematical subjects

- Linear optimization models, modelling, theory, solution methods, and sensitivity analysis
- Discrete optimization models, properties, and solution methods
- Optimization problems that can be modelled as flows in networks, theory, and solution methods
- Multi-objective optimization
- Mixes of the above
- Overview of non-linear optimization models, properties, and solution methods

Activities

- Applications of optimization
- Mathematical modelling
- Theory – mathematical properties of the models
- Solution techniques – algorithms
- Software solvers
- Implementation of models in solvers
- Analysis of results

Optimization: "Do something as good as possible"

- **Something:** Which are the decision alternatives?
  - $x_{ij} = \begin{cases} 1 & \text{if customer } j \text{ is visited } \textit{directly} \text{ after customer } i \\ 0 & \text{else} \end{cases}$
  - $y_{ik} = \begin{cases} 1 & \text{if customer } i \text{ is visited by vehicle } k \\ 0 & \text{else} \end{cases}$
- **Possible:** What restrictions are there?
  - Each customer should be visited exactly once
  - Time windows, transport needs and capacity, pick up at one customer – deliver at another, different types of vehicles, ...
- **Good:** What is a relevant optimization criterion?
  - Minimize the total distance / travel time / emissions / waiting time / ...

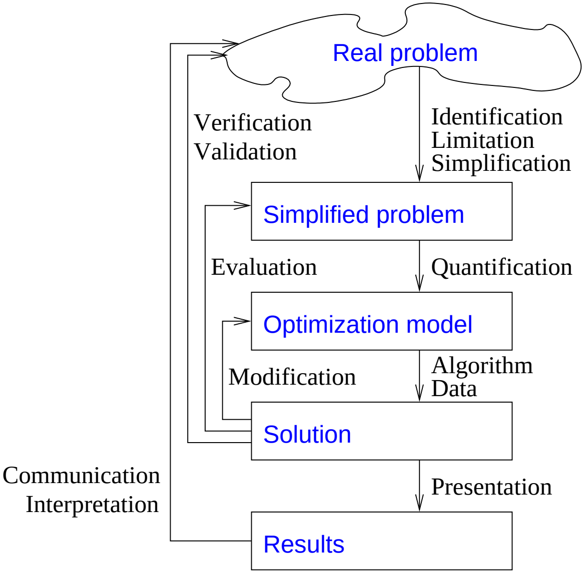
The classical travelling salesperson problem

- Variants of routing problems: refrigerated goods, transportation service for disabled persons, school buses, hybrid propulsion vehicles (electr./diesel), robot operations, ...

A few examples of application areas

- **Logistics: production and transport**
  - Optimize routes for transports, snow removal, school buses, ...
  - Location of stores, warehouses, vehicle fleet parkings, ...
  - Planning of wood cut and transports
  - Packing of containers
  - Production planning and scheduling
  - Dimensioning of batteries and electric motors in routing applications
- **Energy**
  - Energy production planning wrt varying supply and demand
  - Investment in energy production technology
  - Location of power plants and infrastructure
- **Finance**
  - Financial risk management
  - Portfolio optimization
  - Investment planning
- **Medicine**
  - Compute radiation directions/intensities for cancer treatment
  - Reconstruct images from x-ray measurements

The process of optimization

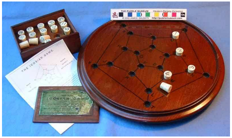


History of mathematical optimization

- During World War II decision problems became systematically treated: *Operations Research*
- After the war: use of operations research for *civil operations*
- The ideas spread to many countries
- Early operations research include *inventory planning*
- This course will treat mathematical optimization models and methods for decision problems that can be modelled using linear forms and continuous and/or integer requirements on the variables

A few moments in optimization history

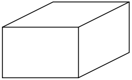
- Euler (1735): Seven bridges of Köningsberg (graph theory)
- Gauss (1777–1855): 1st optimization technique – *steepest descent*
- W.R. Hamilton (1857): "icosian game" $\Rightarrow$ *the travelling salesperson problem* (Hamilton cycle)



- L.V. Kantorovich (1939): A linear model for *optimization of plywood manufacturing* and an *algorithm* for its solution
- George B. Dantzig (1947): Linear programming – *the simplex algorithm* (exponential time)
    - Program $\Leftrightarrow$ military training and logistics schedules
- Kantorovich and Koopmans (1975): Nobel Memorial Prize in Economic Sciences
- N. Karmarkar (1984) algorithm for linear programming (polynomial time)

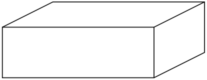
A tiny manufacturing example: Produce tables and chairs from two types of blocks

Small block



$\times 8$

Large block

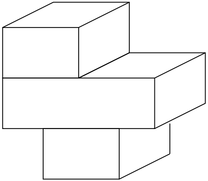


$\times 6$

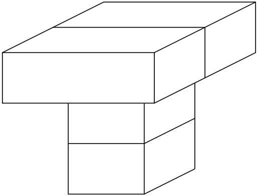
$\Rightarrow$

$\Rightarrow$

Chair



Table



A tiny manufacturing example, continued

- A chair is assembled from one large and two small blocks
- A table is assembled from two blocks of each size
- Only 6 large and 8 small blocks are avaliable
- A table is sold at a revenue of 1600:-
- A chair is sold at a revenue of 1000:-
- Assume that all items produced can be sold and determine an optimal production plan

A tiny mathematical optimization model

- **Something** – What decision alternatives? $\Rightarrow$ Variables

$$x_1 = \text{number of tables produced and sold}$$
$$x_2 = \text{number of chairs produced and sold}$$

- **Possible** – What restrictions? $\Rightarrow$ Constraints
  - Maximum supply of large blocks: 6

$$2x_1 + x_2 \leq 6$$

  - Maximum supply of small blocks: 8

$$2x_1 + 2x_2 \leq 8$$

  - Physical restrictions (also: x_1, x_2 integral)

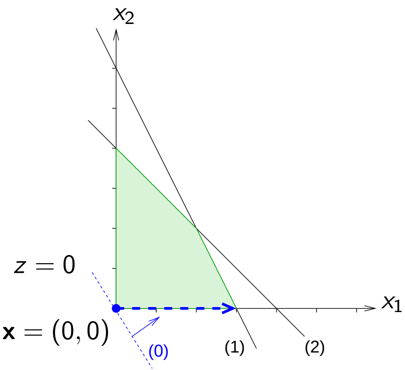
$$x_1, x_2 \geq 0$$

- **Good** – Relevant optimization criterion?$\Rightarrow$Objective function
  - Maximize the total revenue

$$1600x_1 + 1000x_2 \to \max$$

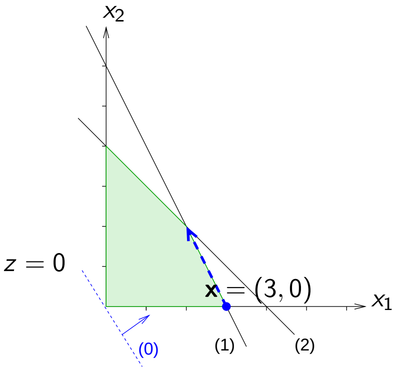
Solve the tiny model using LEGO and marginal values

- Start at no production:
  $x_1 = x_2 = 0$
  Use the "best marginal profit" to choose the item to produce
  - x_1 has the highest marginal profit (1600:-/table)
    $\Rightarrow$ produce as many tables as possible
  - At $x_1 = 3$: no more large blocks left



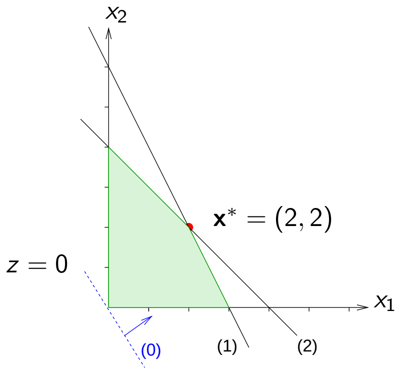
Solve the tiny model using LEGO and marginal values

- The marginal value of x_2 is now 200:- since taking apart one table (-1600:-) yields two chairs ($+2000$:-) $\Rightarrow$ 400:-/2 chairs
  - Increase x_2 maximally $\Rightarrow$ decrease x_1
  - At $x_1 = x_2 = 2$: no more small blocks



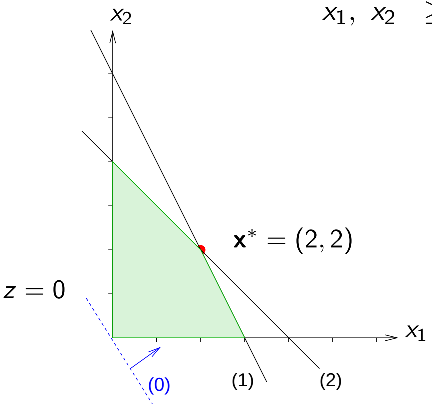
Solve the tiny model using LEGO and marginal values

- The marginal value of x_1 is negative (to build one more table one has to take apart two chairs $\Rightarrow -400$:-)
  The marginal value of x_2 is -600:- (to build one more chair one table must be taken apart)
  $\Longrightarrow$ Optimal solution:
  $x_1 = x_2 = 2$



Geometric solution of the tiny model

$$
\begin{array}{llrcrcrl}
\text{maximize} & z = & 1600x_1 & + & 1000x_2 & & & (0) \\
\text{subject to} & & 2x_1 & + & x_2 & \leq & 6 & (1) \\
& & 2x_1 & + & 2x_2 & \leq & 8 & (2) \\
& & & & x_1, \; x_2 & \geq & 0 &
\end{array}
$$



Linear optimization models (programs)

- The tiny manufacturing model is a *linear program* (LP), i.e., all relations are described by *linear forms*
- A general linear program:

$$\left[\begin{array}{llll} \text{minimize } \textit{or} \text{ maximize} & c_1x_1 + \ldots + c_nx_n & & \\ \text{subject to} & a_{i1}x_1 + \ldots + a_{in}x_n & \left\{\begin{array}{c} \leq \\ = \\ \geq \end{array}\right\} & b_i, \quad i = 1, \ldots, m \\ & x_j & \geq & 0, \quad j = 1, \ldots, n \end{array}\right]$$

- The non-negativity constraints on x_j, $j = 1, \ldots, n$ are not necessary, but usually assumed (transformation always possible)
- An optimal extreme point of a (feasible and bounded) linear program can always be found in an extreme point of the feasible set

Integer linear optimization models

- In the tiny manufacturing model, the numbers of chairs and tables *must* be integers
- The data in this example is "cleared up" such that the *extreme points of the feasible set have integer values*
- What if the *extreme points have fractional values*?

$$\begin{array}{lrcrcrcl} \text{maximize} & z & = & 1600x_1 & + & 1000x_2 & & \\ \text{subject to} & & & 2x_1 & + & x_2 & \leq & 5 \\ & & & 2x_1 & + & 2x_2 & \leq & 9 \\ & & & & & x_1, \ x_2 & \geq & 0 \end{array}$$

$\Rightarrow \quad x_1^* = \frac{1}{2}, x_2^* = 4 \quad \Rightarrow \quad$ Integer requirements are needed

$\Rightarrow$ Integer linear optimization models and methods

Discrete/integer/binary modelling

- A variable is called *discrete* if it can take only a countable set of values, e.g.,
  - Continuous variable: $x \in [0, 8] \iff 0 \leq x \leq 8$
  - Discrete variable: $x \in \{0, 4.4, 5.2, 8.0\}$
  - *Integer* variable: $x \in \{0, 1, 2, \ldots\}$
- A *binary* variable can only take the values 0 or 1, i.e., all or nothing

  E.g., a wind-mill can produce electricity only if it is built
  - Let $y = 1$ if the mill is built, otherwise $y = 0$
  - Capacity of a mill: C
  - Production $x \leq Cy$ (also limited by wind force etc.)
- In general, models with only continuous variables are more tractable than models with integrality/discrete requirements on the variables, but there are important exceptions!
- More about this to come in this course