MVE165/MMG631 Linear and integer optimization with applications Lecture 8b Maintenance scheduling optimization; Assignment 2

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Maintenance in industry

- *Optimal maintenance* = obtain reliability at the least cost
- *Maintenance* costs/year:
  14000 billion SEK (in EU), 275 billion SEK (in Sweden)
- *Maintenance* is often seen merely as a cost
- *Maintenance* is sometimes done too often—inspections and measurements may damage the systems
- Sometimes—like with road/rail infrastructure and "Miljonprogramhusen'—it is performed seldom
- Truth: well performed *maintenance* is an investment in availability and safety

Maintenance principles

- Preventive maintenance (PM): actions that prevent failure
- Corrective maintenance (CM): actions after failure, repairs
- Condition based maintenance (CBM): measurements $\rightarrow$ predictions $\rightarrow$ actions according to a maintenance principle
- Opportunistic maintenance (OM): when maintenance must be performed, make also some (additional) preventive maintenance actions

A simple example, I

A system with n components

- Life of component i: T_i time units (intervals)
- Time horizon: T time units (e.g. contract period)
- Cost of a spare component of type i at time t: c_{it} monetary units
- Cost for performing any maintenance at time t: d_t monetary units

A simple example, II

Variables are logical – do something or not

The model uses binary variables:

$$x_t = \begin{cases} 1, & \text{if “something” is performed at time } t \\ 0, & \text{otherwise} \end{cases}$$

A decision often implies other necessary decisions

- Example: if component i shall be replaced at time t maintenance must be performed
- Such logical relations are equivalent to linear constraints:

$$\text{if A then B} \qquad \Longleftrightarrow \qquad x_A \leq x_B$$

The basic replacement problem, I

Goal: minimize the total cost for keeping the system working during the contract period:

Mathematical model

$$
\begin{aligned}
\underset{(x,z)}{\text{minimize}} \quad & \sum_{t=1}^{T} \left(\sum_{i=1}^{N} c_{it} x_{it} + d_t z_t \right), && && \text{(1a)} \\
\text{subject to} \quad & \sum_{t=\ell+1}^{\ell+T_i} x_{it} \geq 1, && \ell = 0, \dots, T - T_i,\ i = 1, \dots, N, && \text{(1b)} \\
& x_{it} \leq z_t, && t = 1, \dots, T,\ i = 1, \dots, N, && \text{(1c)} \\
& x_{it} \geq 0, && t = 1, \dots, T,\ i = 1, \dots, N, && \text{(1d)} \\
& z_t \leq 1, && t = 1, \dots, T, && \text{(1e)} \\
& x_{it}, z_t \in \{0, 1\}, && t = 1, \dots, T,\ i = 1, \dots, N && \text{(1f)}
\end{aligned}
$$

The basic replacement problem, II

Objective (1a)

Minimize the total cost of having a working system during the contract period

$$\text{minimize}_{(x,z)} \sum_{t=1}^{T} \left(\sum_{i=1}^{N} c_{it} x_{it} + d_t z_t \right)$$

Constraint (1b)

For any given item i in the system, the component must be replaced at some point during *every* time interval of T_i time steps

$$\sum_{t=\ell+1}^{\ell+T_i} x_{it} \geq 1, \quad \ell = 0, \dots, T - T_i, \; i = 1, \dots, N$$

The basic replacement problem, III

Constraint (1c)

No replacement can be performed at time t without paying the fixed cost d_t (for a maint. operation); once we pay, any maint. action becomes possible (at no extra fixed cost) at time t

$$x_{it} \leq z_t, \quad t = 1, \ldots, T, \; i = 1, \ldots, N$$

Constraints (1d)–(1f)

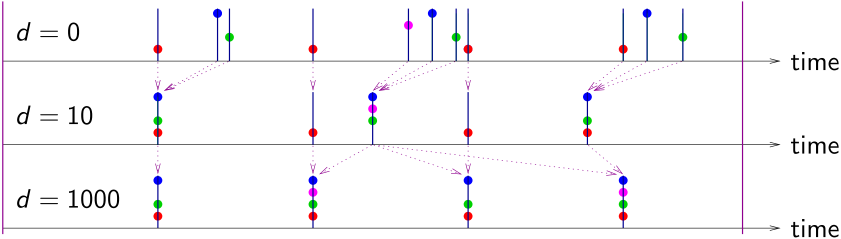
Ensure that the variables take only meaningful values

$$x_{it} \geq 0, \; z_t \leq 1, \; x_{it}, z_t \in \{0, 1\}, \quad t = 1, \ldots, T, \; i = 1, \ldots, N$$

Opportunistic maintenance or not?

Example: four components with different prices and lives

- A *replacement* is marked with a dot; its colour represents the type of component replaced
- The larger the fixed cost, the more beneficial *opportunistic* maintenance becomes; also more items are replaced



Constraint structure—example

Time horizon: $T = 8$. Component #3: $T_3 = 4$

$$\sum_{t=\ell+1}^{\ell+T_3} x_{3t} \geq 1, \qquad \ell = 0, \ldots, T - T_3$$

$$\iff \quad \sum_{t=\ell+1}^{\ell+4} x_{3t} \geq 1, \qquad \ell = 0, \ldots, 4$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_{31} \\ x_{32} \\ \vdots \\ x_{38} \end{bmatrix} \geq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Property I: the replacement problem is NP-hard

Theorem

Set covering is polynomially reducible to the replacement problem

- This essentially means that we *cannot* expect to find an optimal solution in a time that is proportional to a polynomial function of the problem size (i.e., $T(N+1)$ variables and $\approx 4NT$ constraints)
- Basic complexity theory: Chapter 2.6 in the course book

Property II: with fixed z the problem over x is easy

- The constraint matrix of (1b), (1d) (concerning only the variables x) has the "consecutive ones" property (cf. totally unimodular matrix)

$\Rightarrow$ For fixed values of z, the problem over x can be solved as a *linear program*

- For each i, the linear programming dual problem can be solved by a "greedy" algorithm $\Rightarrow$ primal solution by complementarity; see [a], Algorithm 1, page 297
- The latter is typically 5–40 times faster than solving as a general linear program, and 25–400 times faster when costs are monotone with time (i.e., $\forall t$ either $c_{it} \leq c_{i,t+1}$ or $c_{it} \geq c_{i,t+1}$) holds; see [a], Algorithm 2, page 299

[a] T. Almgren, N. Andréasson, M. Patriksson, A.-B. Strömberg, A. Wojciechowski, M. Önnheim (2012): *The opportunistic replacement problem: theoretical analyses and numerical tests*, Mathematical Methods of Operations Research, 76(3) pp. 289–319. http://link.springer.com/article/10.1007%2Fs00186-012-0400-y

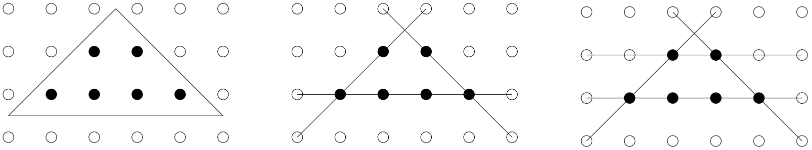
Property III: all inequalities define facets*

An ideal formulation: the convex hull of all feasible integer points

$$X = \left\{ \mathbf{x}^1, \dots, \mathbf{x}^P \right\}, \text{ where } \mathbf{x}^p \text{ are integer vectors}$$

$$X^{\text{conv}} = \left\{ \sum_{p=1}^{P} \mathbf{x}^p \lambda_p \,\middle|\, \sum_{p=1}^{P} \lambda_p = 1;\ \lambda_p \geq 0,\ p = 1, \dots, P \right\}$$

No inequalities define facets | All inequalities define facets | An integral polyhederon (ideal)



* See [a], Section 5.1–5.2

A generalized model

New variable definition

Define the set

$$\mathcal{I} := \{ (s,t) \mid 0 \le s < t \le T+1;\ s,t \in Z \}$$

of *replacement intervals* and introduce the variables

$$x^i_{st} = \begin{cases} 1, & \text{if component } i \text{ receives PM at the} \\ & \text{times } s \text{ and } t, \text{ and } \textit{not} \text{ in-between,} \\ 0, & \text{otherwise,} \end{cases} \qquad \begin{array}{l} i \in \mathcal{N}, \\ (s,t) \in \mathcal{I}, \end{array}$$

and

$$z_t = \begin{cases} 1, & \text{if maintenance occurs at time } t, \\ 0, & \text{otherwise,} \end{cases} \qquad t \in \mathcal{T}.$$

A generalized model

$$
\begin{aligned}
\text{minimize} \quad & \sum_{t\in\mathcal{T}} d_t z_t + \sum_{i\in\mathcal{N}} \sum_{(s,t)\in\mathcal{I}} c^i_{st} x^i_{st}, && && \text{(2a)}\\
\text{subject to} \quad & \sum_{s=0}^{t-1} x^i_{st} \le z_t, && i\in\mathcal{N},\ t\in\mathcal{T}, && \text{(2b)}\\
& \sum_{s=0}^{t-1} x^i_{st} = \sum_{r=t+1}^{T+1} x^i_{tr}, && i\in\mathcal{N},\ t\in\mathcal{T}, && \text{(2c)}\\
& \sum_{t=1}^{T+1} x^i_{0t} = 1, && i\in\mathcal{N}, && \text{(2d)}\\
& x^i_{st} \in\{0,1\}, && i\in\mathcal{N},\ (s,t)\in\mathcal{I}, && \text{(2e)}\\
& z_t \in\{0,1\}, && t\in\mathcal{T}. && \text{(2f)}
\end{aligned}
$$

Assignment 2: Maintenance Scheduling

Assignment tasks in summary

- Study and compare the ILP models for maintenance planning
- Elaborate with the models - w.r.t. facets, integrality property, time horizon
- Heuristic solutions—local search method
- Add side constraints—modelling additional properties
- Students aiming at grade 4, 5, or VG must answer ALL the questions
- *Deadline* for handing in report: *May 10 at 9:30*
- *Hand in* the same report, individually: *May 10, 12:00–17:00*
- *Deadline* for collaboration report, individually: *May 11–13*
- Written opposition (peer review, individual) on another report. *Deadline: May 14*