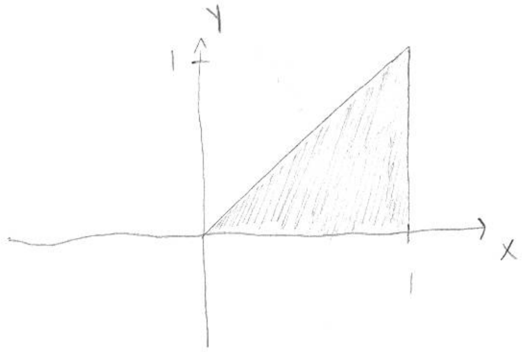
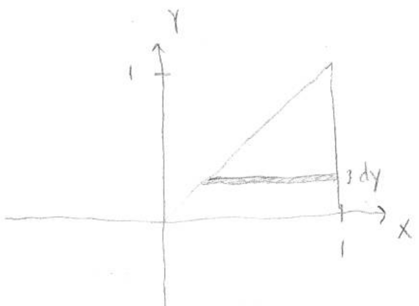
14.2.15. Skissa integrationsområdet och beräkna integralen för $\int_{y=0}^{1}\left(\int_{x=y}^{1} e^{-x^2}dx\right)dy$

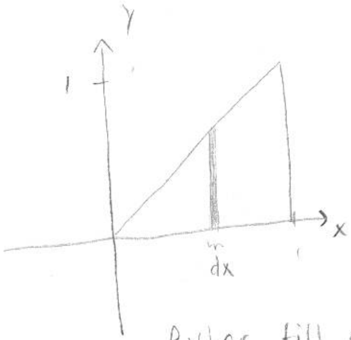
Integrationsområde:



- $\int_{y=0}^{1}\int_{x=y}^{1} e^{-x^2}dx\,dy = \int_{y=0}^{1}\left(\int_{x=y}^{1} e^{-x^2}dx\right)dy = \left\{ \int e^{-x^2}dx \right.$ saknar primitiv, men vi kan byta
- integrationsordning. $\Big\} = \int_{x=0}^{1}\int_{y=0}^{x} e^{-x^2}dy\,dx = \int_{x=0}^{1} e^{-x^2}\left(\int_{y=0}^{x} dy\right)dx = \int_{x=0}^{1} e^{-x^2}\left[y\right]_0^x dx$

$= \int_{x=0}^{1} xe^{-x^2}dx = -\frac{1}{2}\int_{0}^{1} -2xe^{-x^2}dx = -\frac{1}{2}\left[e^{-x^2}\right]_0^1 = -\frac{1}{2}\left(\frac{1}{e}-1\right) = \frac{1}{2}\left(1-\frac{1}{e}\right)$





- ursprunglig integrationsordning ger att vi "summerar" på detta sätt

Byter till att "summera" på detta hållet.

14.3.5. Bestäm om $\iint_Q \frac{x^2+y^2}{(1+x^2)(1+y^2)}dA$ konvergerar eller divergerar, där Q är första kvadranten i xy-planet. Beräkna integralen om den är konvergent.

$$\iint_Q \frac{x^2+y^2}{(1+x^2)(1+y^2)}dA = \int_{y=0}^{\infty}\int_{x=0}^{\infty}\frac{x^2+y^2}{(1+x^2)(1+y^2)}dx\,dy = \lim_{R\to\infty}\lim_{S\to\infty}\int_{y=0}^{R}\int_{x=0}^{S}\frac{x^2+y^2}{(1+x^2)(1+y^2)}dx\,dy$$

där

$$\int_{y=0}^{R}\int_{x=0}^{S} \frac{x^2+y^2}{(1+x^2)(1+y^2)}\,dx\,dy = \int_{y=0}^{R} \frac{1}{1+y^2}\left(\int_{x=0}^{S} \frac{x^2+y^2}{1+x^2}\,dx\right)dy = \int_{y=0}^{R} \frac{1}{1+y^2}\left(\int_{x=0}^{S} \frac{x^2}{1+x^2} + \frac{y^2}{1+x^2}\,dx\right)dy$$

$$= \int_{y=0}^{R} \frac{1}{1+y^2}\left(\int_{x=0}^{S} \frac{x^2+1-1}{1+x^2} + \frac{y^2}{1+x^2}\,dx\right)dy = \int_{y=0}^{R} \frac{1}{1+y^2}\left(\int_{x=0}^{S} 1 - \frac{1}{1+x^2} + \frac{y^2}{1+x^2}\,dx\right)dy$$

$$= \int_{y=0}^{R} \frac{1}{1+y^2}\Big[x + (y^2-1)\arctan(x)\Big]_{x=0}^{S} dy = \int_{y=0}^{R} \frac{1}{1+y^2}\big(s + (y^2-1)\arctan(s)\big)\,dy$$

$$= (s-\arctan(s))\int_{y=0}^{R} \frac{1}{1+y^2}\,dy + \arctan(s)\int_{y=0}^{R} \frac{y^2}{1+y^2}\,dy = (s-\arctan(s))\Big[\arctan y\Big]_0^R$$

$$+ \arctan(s)\Big[y - \arctan y\Big]_0^R = (\underbrace{s}_{\to\infty} - \underbrace{\arctan(s)}_{\to\pi/2})\underbrace{\arctan(R)}_{\to\pi/2} + (\underbrace{R}_{\to\infty} - \underbrace{\arctan(R)}_{\to\pi/2})\underbrace{\arctan(s)}_{\to\pi/2} \longrightarrow \infty$$

då $R, S \to \infty \Rightarrow$ <u>integralen divergerar</u>

14.3.7. Beräkna $\iint_{\mathbb{R}^2} e^{-(|x|+|y|)}\,dA$ eller visa att den divergerar.

$$\iint_{\mathbb{R}^2} e^{-(|x|+|y|)}\,dA = \int_{y=-\infty}^{\infty}\int_{x=-\infty}^{\infty} e^{-(|x|+|y|)}\,dx\,dy = \int_{y=-\infty}^{\infty}\int_{x=-\infty}^{\infty} e^{-|x|}\cdot e^{-|y|}\,dx\,dy$$

$$= \int_{y=-\infty}^{\infty} e^{-|y|}\left(\int_{x=-\infty}^{\infty} e^{-|x|}\,dx\right)dy = \left\{ \begin{array}{l} |x| = |-x| \Rightarrow \text{samma bidrag till integralen på} \\ (-\infty, 0) \text{ som } (0,\infty) \end{array} \right\}$$

$$= \int_{y=-\infty}^{\infty} e^{-|y|}\left(2\int_{x=0}^{\infty} e^{-x}\,dx\right)dy = \int_{y=-\infty}^{\infty} 2e^{-|y|}\Big[-e^{-x}\Big]_0^{\infty} dy = 2\int_{y=-\infty}^{\infty} e^{-|y|}\cdot 1\,dy$$

$$= 4\int_{0}^{\infty} e^{-y}\,dy = 4\Big[-e^{-y}\Big]_0^{\infty} = \underline{\underline{4}}$$