Idag: Intro
Funktioner 1.1
Visualisering (datorövning)

Intro Flervariabelanalys

* Funktion av flera variabler:
  - med vektorbeteckning
    $f(\mathbf{r}) = f(x,y,z)$, $\mathbf{r} = xe_1 + ye_2 + ze_3$
  - med matrisbeteckning
    $f(x) = f(x_1, \dots, x_n)$, $x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$
* Vektorvärd funktion:
  - $\mathbf{r}(t) = x(t)e_x + y(t)e_y + z(t)e_z$
  - $\mathbf{F}(\mathbf{r}) = F_x(\mathbf{r})e_x + F_y(\mathbf{r})e_y + F_z(\mathbf{r})e_z$
  - $f(x) = \begin{bmatrix} f_1(x_1, \dots, x_n) \\ \vdots \\ f_m(x_1, \dots, x_n) \end{bmatrix}$

* Derivera: Df, ∇f, $\nabla \cdot F$, $\nabla \times \mathbf{F}$
* Integrera: $\iiint_D f \, dx\,dy\,dz$
* Ekv. system: $f(x) = 0$, Newtons metod
* Partiella differentialekvationer (PDE)
  $-\nabla \cdot (a \nabla u) = f$
* Finita elementmetoden (FEM)

Funktioner 1.1

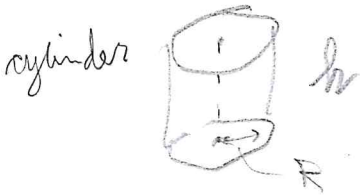
Def 1.1 $f: X \to Y$
$x \mapsto y = f(x)$

X domän $D(f) \subseteq X$
Y kodomän $R(f) \subseteq Y$

I kp 1-2: $f: \mathbb{R} \to \mathbb{R}$
linjär algebra: $f(x) = Ax$, $A \in \mathbb{R}^{m \times n}$ (linjär funktion), $f: \mathbb{R}^n \to \mathbb{R}^m$

Nu: allmän funktion $f: \mathbb{R}^n \to \mathbb{R}^m$

Ex 1.2



$V = \pi R^2 h$

$f: \mathbb{R}^2 \to \mathbb{R}$

$f(x,y) = \pi x^2 y$

$f(x) = \pi x_1^2 x_2, \quad x = (x_1, x_2)$

$\mathcal{D}(f) = \mathbb{R}_+ \times \mathbb{R}_+ \subset X = \mathbb{R}^2$ $(R \geq 0, h \geq 0)$

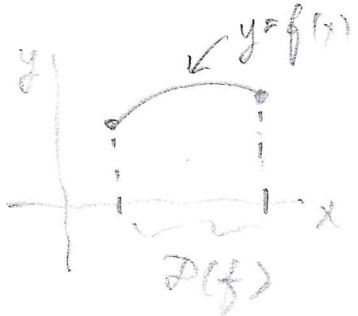
$\mathcal{R}(f) = \mathbb{R}_+ \subset Y = \mathbb{R}$ $(V \geq 0)$

Obs: ofta svårt bestämma $\mathcal{R}(f)$, men räcker att bestämma Y.

Ex Graf, $f: \mathbb{R} \to \mathbb{R}$

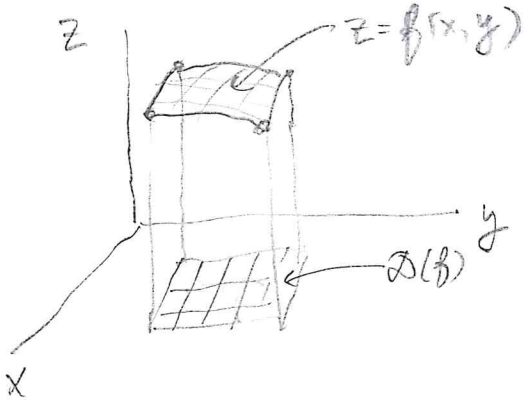
$\{(x,y): y = f(x)\} \subseteq \mathbb{R}^2$

kurva i planet



Ex Graf, $f: \mathbb{R}^2 \to \mathbb{R}$

$\{(x,y,z): z = f(x,y)\} \subseteq \mathbb{R}^3$



yta i rummet

Ex Nivåkurvor, $f: \mathbb{R}^2 \to \mathbb{R}$

$\{(x,y): f(x,y) = c\} \subseteq \mathbb{R}^2$

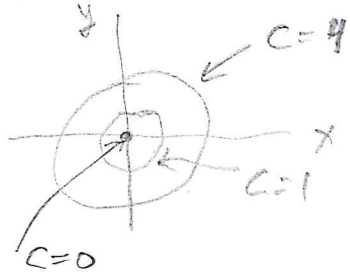
för olika värden på c.

$x^2 + y^2 = c$

$c > 0$ cirkel radie $\sqrt{c}$

$c = 0$ origo

$c < 0$ $\emptyset$

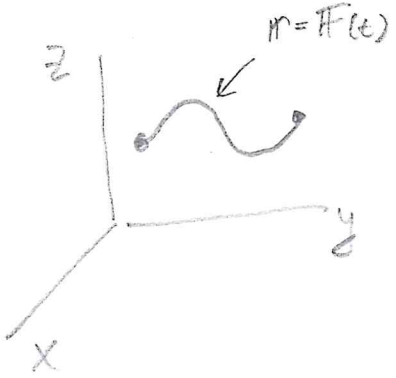


Ex Kurva, $F: \mathbb{R} \to \mathbb{R}^3$

$r = F_x(t)e_x + F_y(t)e_y + F_z(t)e_z$, $r = F(t)$

$$\begin{cases} x = F_x(t), \\ y = F_y(t), \\ z = F_z(t), \end{cases} \quad t \in [a,b]$$

kurva på parameterform



Ex Nivåytor, $f: \mathbb{R}^3 \to \mathbb{R}$

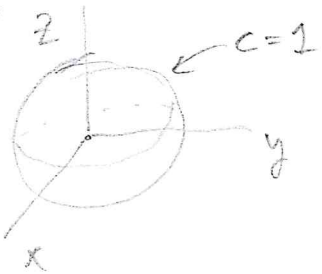
$\{(x,y,z) : f(x,y,z) = C\}$

$x^2 + y^2 + z^2 = C$

$C > 0$ sfär radie $\sqrt{C}$

$C = 0$ origo

$C < 0$ $\emptyset$



Datorövning: visualisering

Kurva i rummet

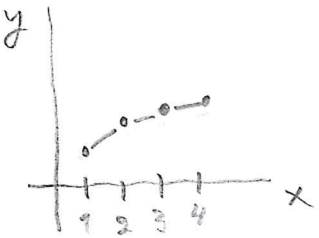
$r = F(t)$, $t \in [a,b]$

```
>> t = linspace(0,2);   mät i [0,2]
>> x = t; y = t.^2; z = t.^3;
>> plot3(x,y,z)
>> grid on; axis equal;
```

Graf (envariabel-funktion)

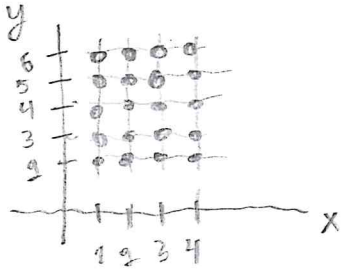
$y = f(x)$, $x \in [a,b]$

```
>> x = 1:4;   mät i [1,4]
>> y = sqrt(x);
>> plot(x,y)
>> x = linspace(1,4);   finare mät
```



Graf (tvåvariabel-funktion)

$z = f(x,y)$




```
>> x = 1:4 (mät i x)
>> y = 2:6 (mät i y)
>> [X, Y] = meshgrid(x,y); (mät i [1,4]×[2,6])
>> Z = X.*Y.^2   (z = xy^2)
>> mesh(X,Y,Z)
>> surf(X,Y,Z)
```

Finare mät: x = linspace(1,4)

$$X = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix} (5\times 4) \qquad Y = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 \\ 6 & 6 & 6 & 6 \end{bmatrix} (5\times 4)$$

Nivåkurvor

$f(x,y) = c$

```
>> contour(X,Y,Z)
>> surfc(X,Y,Z)
```

Nivåytor

$f(x,y,z) = c$

```
>> [X,Y,Z] = meshgrid(x,y,z);
>> V = f(X,Y,Z);
>> isosurface(X,Y,Z,V,0)
```

Visualisering: svårt, kräver övning.

(för smått → syns ej)

(för stort → döljer smått)