Sammanfattning 2: alla dessa integraler.

Obs: samma upplägg för alla tre; enkel-, dubbel- och trippelintegral

1. Enkelintegral

$$\int_I f\,dx = \int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \sum_{i=1}^{n} f(\tilde{x}_i)\,\Delta x_i$$

Variabelbyte: $x = g(u)$, $a = g(A)$, $b = g(B)$

$dx = g'(u)\,du$

$$\int_a^b f(x)\,dx = \int_A^B f(g(u))\,g'(u)\,du$$

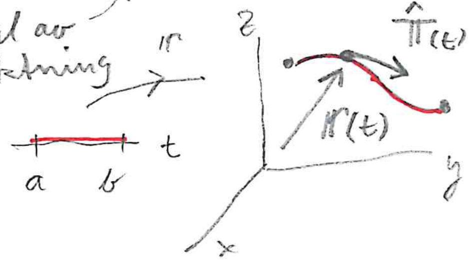
Fundamentalsatsen: $\int_a^b Df(x)\,dx = \Big[f(x)\Big]_a^b$

Kurva: $C: \mathbf{r} = \mathbf{r}(t),\ t \in [a,b]$

C slät om kurvan har tangent. Det finns tangent om $\mathbf{r}'(t) \neq \mathbf{0}$. Då är $\mathbf{T}(t) = \pm\,\mathbf{r}'(t)$ och

$$\hat{\mathbf{T}}(t) = \frac{\mathbf{T}(t)}{\|\mathbf{T}(t)\|}$$

val av riktning



Kurvintegral:

$ds = \|\mathbf{r}'(t)\|\,dt$

$$\int_C f\,ds = \int_a^b f(\mathbf{r}(t))\,\|\mathbf{r}'(t)\|\,dt$$

Längd: $L = \int_C ds$

Tangentkurvintegral: $d\mathbf{r} = \hat{\mathbf{T}}\,ds = \mathbf{r}'(t)\,dt$

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_C \mathbf{F}\cdot\hat{\mathbf{T}}\,ds = \pm\int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\,dt$$

Tolkning: arbete. Tangentkomponenten av $\mathbf{F}$ är $\mathbf{F}\cdot\hat{\mathbf{T}}$.

Fundamentalsats:

om $\mathbf{F} = \nabla\phi$ så är $\int_C \mathbf{F}\cdot d\mathbf{r} = \Big[\phi(\mathbf{r}(t))\Big]_a^b$

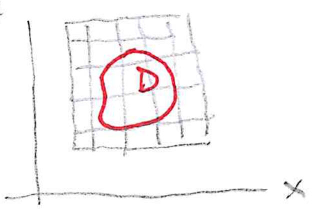
2. Dubbelintegral.

$$\iint_D f\,dA =$$

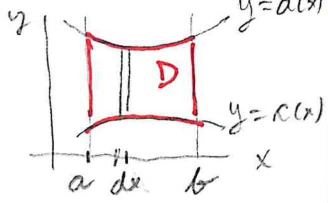
$$= \iint_D f(x,y)\,dx\,dy = \lim_{h\to 0} \sum_{i=1}^{n} \sum_{j=1}^{m} f(\tilde{x}_i, \tilde{y}_j)\,\Delta x_i\, \Delta y_j$$

Fubini: $R=[a,b]\times[c,d]$

$$\iint_R f\,dA = \int_a^b \Big(\int_c^d f(x,y)\,dy \Big)\,dx = \int_c^d \Big(\int_a^b f(x,y)\,dx \Big)\,dy$$



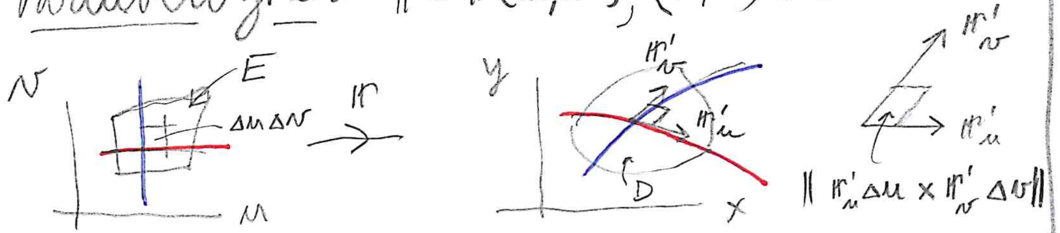
$$\iint_D f\,dA = \int_a^b \Big(\int_{c(x)}^{d(x)} f(x,y)\,dy \Big)\,dx$$



(enkel i y)

Area: $\text{area}(D) = \iint_D dA$

Variabelbyte: $\mathbf{r} = \mathbf{r}(u,v)$, $(u,v)\in E$

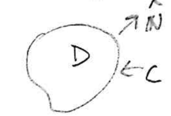


Koordinatkurvor, tangenter, Jacobi-determinant:

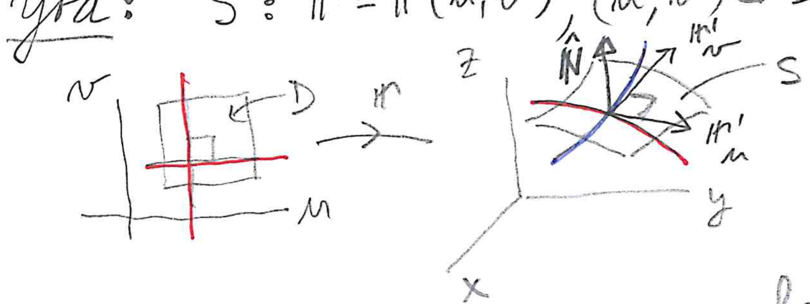
$$dA = \|\mathbf{r}'_u \times \mathbf{r}'_v\|\,du\,dv = \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$$

$$\iint_D f\,dA = \iint_E f(\mathbf{r}(u,v))\,\|\mathbf{r}'_u \times \mathbf{r}'_v\|\,du\,dv$$

Fundamentalsats: Gauss sats i planet. $\iint_D \nabla\cdot\mathbf{F}\,dA = \int_C \hat{\mathbf{N}}\cdot\mathbf{F}\,d$



Yta: $S:\ \mathbf{r}=\mathbf{r}(u,v)$, $(u,v)\in D$



Koordinatkurvor, tangenter.

S är slät om $\exists$ normalvektor.

Det finns om $\mathbf{r}'_u \times \mathbf{r}'_v \neq \mathbf{0}$. Då är

$\mathbf{N} = \pm(\mathbf{r}'_u \times \mathbf{r}'_v)$, $\hat{\mathbf{N}} = \dfrac{\mathbf{N}}{\|\mathbf{N}\|}$ ($\pm$ val av orientering)

Ytintegral: $dS = \|\mathbf{r}'_u \times \mathbf{r}'_v\|\,du\,dv$

$$\iint_S f\,dS = \iint_D f(\mathbf{r}(u,v))\,\|\mathbf{r}'_u \times \mathbf{r}'_v\|\,du\,dv$$

Area: $\text{area}(S) = \iint_S dS$

Normalytintegral:

$$d\mathbf{S} = \hat{\mathbf{N}}\, dS = \pm(\mathbf{r}'_u \times \mathbf{r}'_v)\, du\, dv$$

$$\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_S \mathbf{F}\cdot\hat{\mathbf{N}}\, dS =$$

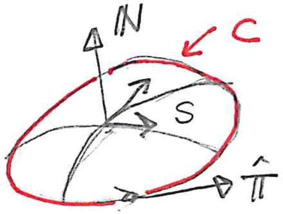
$$= \pm \iint_D \mathbf{F}(\mathbf{r}(u,v))\cdot(\mathbf{r}'_u \times \mathbf{r}'_v)\, du\, dv$$

Tolkning: flöde.

Normalkomponenten av $\mathbf{F}$ är $\mathbf{F}\cdot\hat{\mathbf{N}}$.

Fundamentalsats: Stokes sats

$$\iint_S (\nabla\times\mathbf{F})\cdot\hat{\mathbf{N}}\, dS = \int_C \mathbf{F}\cdot\hat{\mathbf{T}}\, ds$$



3. Trippelintegral

$$\iiint_D f\, dV = \iiint_D f(x,y,z)\, dx\, dy\, dz =$$

$$= \lim_{h\to 0} \sum_i \sum_j \sum_k f(\tilde{x}_i, \tilde{y}_j, \tilde{z}_k)\, \Delta x_i\, \Delta y_j\, \Delta z_k$$

Volym: $\text{vol}(D) = \iiint_D dV$.

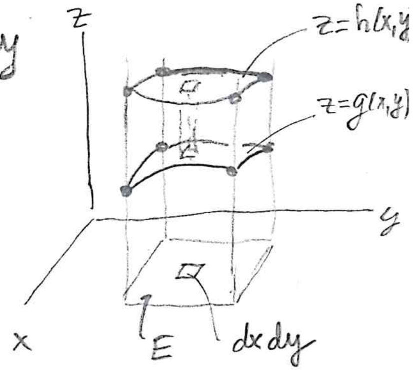
Fubini: $R = [a,b]\times[c,d]\times[g,h]$

$$\iiint_D f\, dV = \int_a^b \left(\int_c^d \left(\int_g^h f(x,y,z)\, dz \right) dy \right) dx$$

i godtycklig ordning

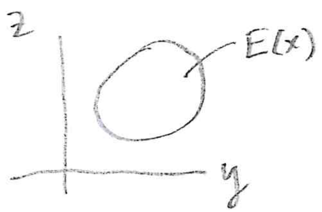
Enkelt i z: $g(x,y) \le z \le h(x,y),\ (x,y)\in E$

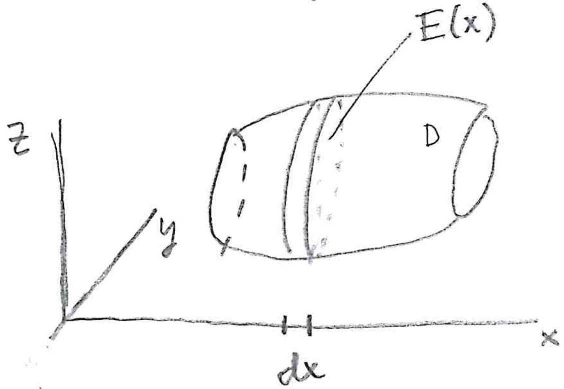
$$\iiint_D f\, dV = \iint_E \left(\int_{g(x,y)}^{h(x,y)} f(x,y,z)\, dz \right) dx\, dy$$



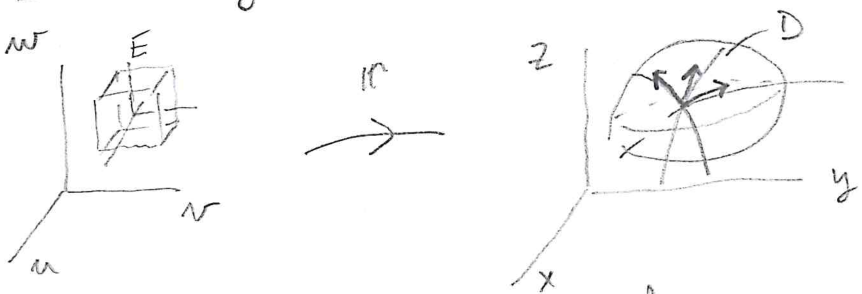
Skivning:

$$\iiint_D f\,dV = \int_a^b \left(\iint_{E(x)} f(x,y,z)\,dy\,dz \right) dx$$





Variabelbyte: $\mathbf{r} = \mathbf{r}(u,v,w)$, $(u,v,w) \in E$



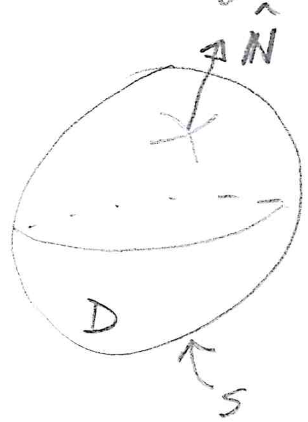
Koordinatkurvor, tangenter, Jacobi-determinant.

$$dV = |(\mathbf{r}'_u \times \mathbf{r}'_v)\cdot \mathbf{r}'_w|\,du\,dv\,dw = \left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right| du\,dv\,dw$$

$$\iiint_D f\,dV = \iiint_E f(\mathbf{r}(u,v,w))\,|(\mathbf{r}'_u \times \mathbf{r}'_v)\cdot \mathbf{r}'_w|\,du\,dv\,dw$$

Fundamentalsats: Gauss divergenssats

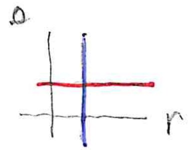
$$\iiint_D \nabla\cdot\mathbf{F}\,dV = \iint_S \hat{\mathbf{N}}\cdot\mathbf{F}\,dS$$

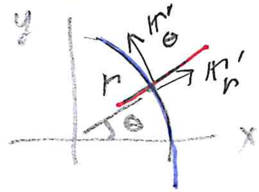


Polära koordinater (r,θ)

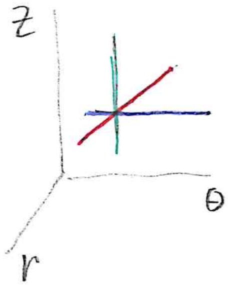
$$\begin{cases} x = r\cos(\theta) \\ y = r\sin(\theta) \end{cases}$$

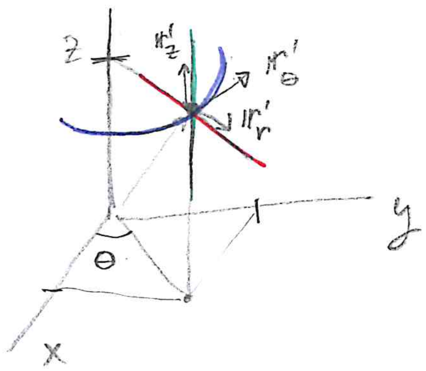
$$dA = r\,dr\,d\theta$$



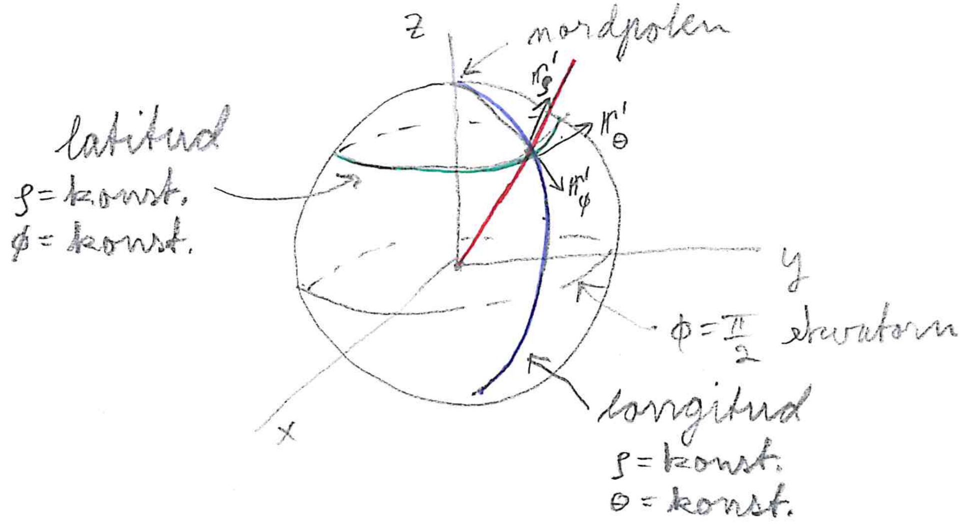


Cylindriska koordinater (r, θ, z)





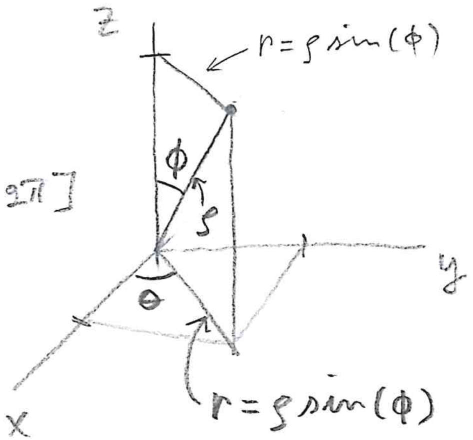
$$\begin{cases} x = r\cos(\theta) \\ y = r\sin(\theta) \\ z = z \end{cases} \qquad dV = r^2\,dr\,d\theta\,dz$$



Sfäriska koordinater (ρ, ϕ, θ)

$$\begin{cases} x = \rho\sin(\phi)\cos(\theta) \\ y = \rho\sin(\phi)\sin(\theta) \\ z = \rho\cos(\phi) \end{cases}$$

$\rho \in [0, \infty)$, $\phi \in [0, \pi]$, $\theta \in [0, 2\pi]$



Graf $y = f(x)$

$$\begin{cases} x = t \\ y = f(t) \end{cases} \qquad ds = \sqrt{1 + f'(t)^2}\,dt$$

Graf $z = f(x, y)$

$$\begin{cases} x = u \\ y = v \\ z = f(u,v) \end{cases} \qquad dS = \sqrt{1 + f_1'(u,v)^2 + f_2'(u,v)^2}\,du\,dv$$

Lycka till med tentan!

/Stig