

MVE270 Tentamen 9/4 - 16

Svar eller lösninga

$$\textcircled{1} \quad \nabla f = \left(\sin(2x+3y) + 2x \cos(2x+3y), 3x \cos(2x+3y) \right)$$

$$= (1, 0) \text{ i } \left(\frac{\pi}{4}, 0\right)$$

$$\text{Så } f_{\vec{v}} = (1, 0) \cdot \frac{(1, 2)}{\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$\textcircled{2} \quad f'_\theta = -r \sin \theta f'_x + r \cos \theta f'_y = xf'_y - yf'_x = 0$$

$$\Rightarrow f > h(r) = \hat{h}(x^2+y^2)$$

$$\textcircled{3} \quad \operatorname{div} F = 2z; \quad \text{därigenom sätter vi}$$

$$\iiint_{x^2+y^2+z^2 \leq 1} F \cdot N \, dS = \iiint_{x^2+y^2+z^2 \leq 1} 2z \, dV = 0$$

(4) Kompakt umrunden, das

$$\left\{ \begin{array}{l} \left| \begin{array}{cc} 3x^2 & 3y^2 \\ 4x^3 & 4y^3 \end{array} \right| = 0 \\ x^4 + y^4 = 1 \end{array} \right. \quad \left\{ \begin{array}{l} xy(y-x) = 0 \\ x^4 + y^4 = 1 \end{array} \right.$$

$$\max x = 2 \cdot 2^{-\frac{1}{4}} = \sqrt[4]{2}$$

Swar:

$$\min x = -\sqrt[4]{2}$$

(5)

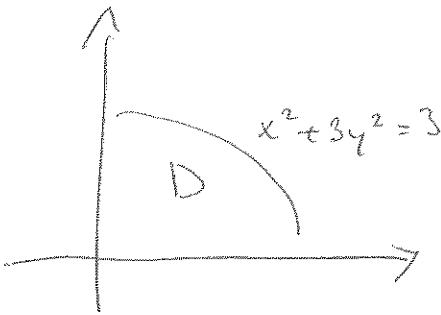
Greens formel gilt

$$\oint_C F dr = \iint_{x^2+y^2 \leq 4} x^2 + y^2 dx dy = 8\pi$$

(6)

Se bilden

(7)

 $f > 0$ i Doch $f \rightarrow \infty$ utanför $f(x, y) \rightarrow -\infty$ då $x \rightarrow \infty$ så minst sätmas

Största värde tror i en stenhård punkt i D

$$\begin{cases} f_x = 3y(1-x^2-y^2) \\ f_y = x(3-x^2-3y^2) \end{cases} \Rightarrow \text{Så}$$

$$\begin{cases} f'_x = 0 \\ f'_y = 0 \end{cases} \Leftrightarrow \begin{cases} x^2 = \frac{3}{4}u \\ y^2 = \frac{1}{4}u \end{cases} \text{ om u}$$

ignorer triviale lösningar

Max värde åter i $(x, y) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ Svar: största värde är $\underline{\frac{3\sqrt{3}}{8}}$

minsta värde sätmas

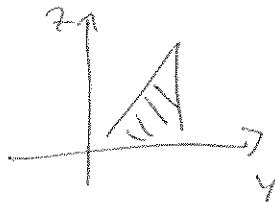
(8)

E begrenzt an planen

$$z = x + y, \quad z = y - x, \quad z = 0 \text{ und } y = 1$$

Größe:

$$\left\{ \begin{array}{l} z - y \leq x \leq y - z \\ 0 \leq z \leq y \\ 0 \leq y \leq 1 \end{array} \right.$$



$$\int_{y-z}^{y-z} y dx = 2y^2 - 2yz, \quad \int_0^y 2y^2 - 2yz dz = y^3$$

och

$$\int_0^1 y^3 dy = \frac{1}{4} \quad \text{Svar} > \frac{1}{4}$$

(9)

$$\text{Låt } \phi = ye^{xz} \text{ så att } \nabla \phi = (ye^{xz}, e^{xz}, xe^{xz})$$

$$\text{Då blir } \int_C ye^{xz} dx + e^{xz} dy + (ye^{xz} + xe^{xz}) dz =$$

$$= \int_C \nabla \phi \cdot dr + \int_C xe^{xz} dz = \phi(2, 3, 1) - \phi(1, 0, 0) + \int_0^1 (t^2 e^t) t^2 \cdot 2t dt =$$

$$= 3e^2 - 0 + 2 \int_0^1 t^5 + t^3 dt = 3e^2 + \frac{5}{6}$$

(10)

$$\mathbf{F} = (x^2y, xy^2, z^2) \Rightarrow \text{rot } \mathbf{F} = (0, 0, y^2 - x^2)$$

Stokes formula: $\int_{\gamma} \mathbf{F} dr = \iint_{\Sigma} \text{rot } \mathbf{F} \cdot \mathbf{N} dS$

der Σ är den del av $x+y+z=1$ som

ligge innanför $x^2+y^2=4$

$$\mathbf{N} dS = (1, 1, 1) dx dy$$

$$\iint_{\Sigma} \text{rot } \mathbf{F} \cdot \mathbf{N} dS = \iint_{x^2+y^2 \leq 4} y^2 - x^2 dx dy = 0$$

