SVAR TENTAMEN 5/4 -2016

1a $\dfrac{x^2\left(\frac{3}{x}-5\right)}{x^2\left(\frac{2}{x^2}+\frac{1}{x}-4\right)} \to \dfrac{+5}{4}$, $x \to \infty$

1b $f' = 9x^2 + \dfrac{4}{x^3}$ $\quad f'(-1) = 5$ $\quad T: y = -6+5(x+1)$ $\quad N: y = -6 - \dfrac{x+1}{5}$

1c $f' = 0 \Leftrightarrow x = -2$ eller $x = 3$

x		-2		3	
f'	$+$	0	$-$	0	$+$
f	↗		↘		↗

avtagande för $x \le -2$ resp $x \ge 3$

1d $f(-0.4) \approx f(0) + \dfrac{f(-1)-f(0)}{-1-0}\cdot(-0.4-0)$

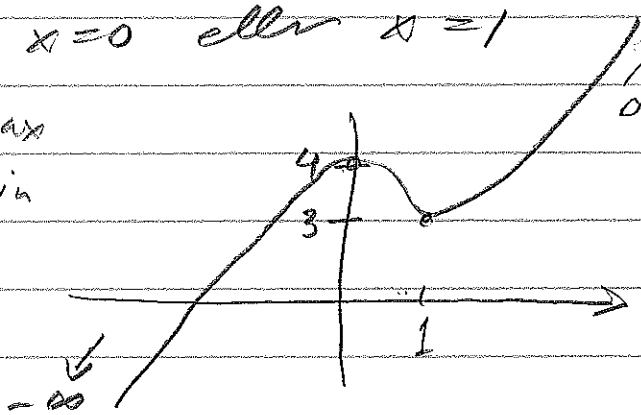
$= -2 + \dfrac{-0.7}{-1}(-0.4) = -2.28$

2a $f' = 6x^2 - 6x = 0 \Leftrightarrow x = 0$ eller $x = 1$

x		0		1	
f'	$+$	0	$-$	0	$+$
f	↗		↘		↗

$x = 0$ lokmax

$x = 1$ lok min



2b $f(-1) = -2$ $\quad f(0) = 5$

$\Rightarrow -1 < \text{rot} < 0$

$x_0 = -1 \Rightarrow x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = -1 - \dfrac{-2}{17} = -0.8824$ $\quad |f(x_1)| = 0.1750$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} = \underline{\underline{-0.87}}$ $\quad |f(x_2)| = 0.0031$

2c $4+3x = 2x^2+5x \Leftrightarrow x = -\frac{1}{2} \pm \sqrt{\frac{1}{4}+2} = -2$ el. 1

~~$\int_{-2}^{1}(4-2x-2x^2)\,dx = \left[4x - x^2 - \frac{2x^3}{3}\right]_{-2}^{1} = 9$~~

(3a) $y' = 3Ae^{3t} - 2Be^{-2t}$ $y'' = 9Ae^{3t} + 4Be^{-2t}$

$y'' - 5y = 4Ae^{3t} - Be^{-2t}$ $A = \frac{1}{2}$ $B = 3$

(3b) $y' = -\frac{1}{3}y$ $y = Ce^{-t/3}$ $y' = -\frac{C}{3}e^{-t/3}$

$y'(0) = -\frac{C}{3} = -2 \Rightarrow C = 6$ $y = 6e^{-t/3}$

(3c) $r = -2 \pm \sqrt{4 - \frac{17}{4}} = -2 \pm \frac{i}{2}$

$y = e^{-2t}(A\cos\frac{t}{2} + B\sin\frac{t}{2})$

$y' = e^{-2t}(-2A\cos\frac{t}{2} - 2B\sin\frac{t}{2} - \frac{A}{2}\sin\frac{t}{2} + \frac{B}{2}\cos\frac{t}{2})$

$2 = A$, $-3 = -2A + \frac{B}{2} \Leftrightarrow B = 2$

(4a) $\begin{cases} x = -1 + 5t \\ y = 2 - 7t \\ z = -3 + 9t \end{cases}$ $t = 2 \Rightarrow (9, -12, 15)$

(4b) $\begin{cases} x + 3z = 0 & (1) \\ 2x - 3y + 5z = 0 & (2) \end{cases}$ t.ex. $z = 1$; $(1) \Rightarrow x = -3$

$(2) \Rightarrow y = \frac{2x + 5z}{3} = \frac{-6+5}{3} = -\frac{1}{3}$

$\Rightarrow$ t.ex. $(-3, -\frac{1}{3}, 1)$ eller $(9, 1, -3)$

(4c) $ax + by + cz = d$ $3x - y - 2z = d$

$d = 3 \cdot 2 - (-4) - 2 \cdot 5 = 0$

(4d) $3 - 2t - 5(2 + 4t) + 4(1 + 3t) = 7$

$-3 - 10t = 7 \Leftrightarrow t = -1 \Rightarrow (5, -2, -2)$