SVAR MVE340 1/6 - 2016

1a) $\dfrac{x^3(3/x^2-1)}{x^3(2+4/x)} \to \dfrac{-1}{2},\ x\to\infty$

1b) $f'(x)=6x-2 \quad f'(-1)=-8$ Tgt: $y=6-8(x+1)$

N: $y=6+\frac{1}{8}(x+1)$

1c)

x		-2		2	
f'	$-$		$+$		$-$
f	$\searrow$		$\nearrow$		$\searrow$

växande för $-2\le x\le 2$

1d) $f(1.4)\approx f(1)+\dfrac{f(2)-f(1)}{2-1}(1.4-1)=-2.3+\dfrac{-3.0+2.3}{1}\cdot 0.4$

$=-2.58$

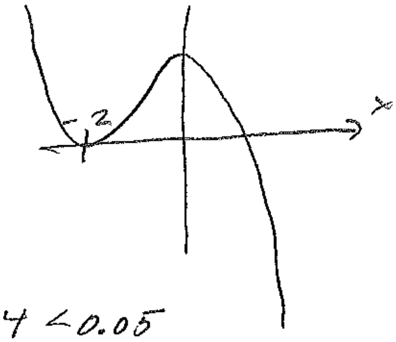
2a) $f'=-3x^2-6x=-3x(x+2)$

x		-2		0	
f'	$-$		$+$		$-$
f	$\searrow$		$\nearrow$		$\searrow$

$x=-2$ lok min

$x=0$ lok max

x	y
-2	0
0	4



2b)

x	$f(x)$
0	4
-1	2
-2	-8

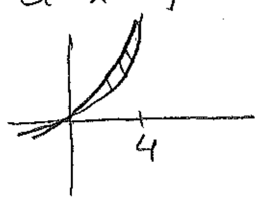
$x_0=-1$

$x_1=-1.4 \quad |f(x_1)|=0.7$

$x_2=-1.319 \quad |f(x_2)|=0.034<0.05$

2c) $x+3x^2=2x^2+5x \iff x^2-4x=0 \iff x=0$ el $x=4$

$\int_0^4 (2x^2+5x-x-3x^2)\,dx=\left[2x^2-\frac{x^3}{3}\right]_0^4=\frac{32}{3}$



3a) $y'=2Ax+B \quad y''=2A$

$2A-5(2Ax+B)=-10Ax+2A-5B$

$A=-1/10 \quad B=-1/25$

3b) $y'=-\frac{1}{2}y \quad y=Ce^{-\frac{1}{2}t} \quad y'=-\frac{1}{2}Ce^{-\frac{1}{2}t} \quad y'(0)=-3$

$\iff -\frac{1}{2}C=-3 \quad C=6 \quad y=6e^{-\frac{1}{2}t}$

3c) $r^2+6r+25=0 \quad r=-3\pm\sqrt{9-25}=-3\pm 4i$

$y=e^{-3t}(C_1\cos 4t+C_2\sin 4t) \quad y(0)=5 \iff 5=C_1\cdot 1$

$y'=e^{-3t}(-3C_1\cos 4t-3C_2\sin 4t-4C_1\sin 4t+4C_2\cos 4t) \quad y'(0)=1 \iff 1=-3C_1+4C_2$

$C_1=5 \quad C_2=4 \quad y=e^{-3t}(5\cos 4t+4\sin 4t)$

4a) $P=P_0+t(P_1-P_0)=(-1,2,0)+t(5,-5,2)$ $\begin{cases} x=-1+5t\\ y=2-5t\\ z=2t\end{cases}$

Annan punkt t.ex. $t=-1 \Rightarrow (-6,5,-2)$

4b)

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A=[1 -3 -1; 4 0 3; -1 4 0]
b=[1; -7; 8]
A\b
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4c $\Sigma x = 15 \quad \Sigma x^2 = 55 \quad \Sigma y = 2.5 \quad \Sigma xy = 26$

$$\left.\begin{aligned} 55k + 15m &= 26 \\ 15k + 5m &= 2.5 \end{aligned}\right\} (-3) \qquad 10k = 18.5 \quad k = 1.85$$

$$m = \frac{2.5 - 15k}{5} = -5.05$$

$$y = 1.85x - 5.05$$

4d

$$\begin{aligned} 1\cdot k + m &= -3.2 \\ 2\cdot k + m &= -1.4 \\ 3\cdot k + m &= 0.7 \\ 4\cdot k + m &= 2.1 \\ 5k + m &= 4.3 \end{aligned} \implies$$

A = [1 1; 2 1; 3 1; 4 1; 5 1]
b = [-3.2; -1.4; 0.7; 2.1; 4.3]
A\b

5 a

$$\int \frac{\sin^2 x}{1+\tan^2 x}\,dx = \int \sin^2 x \cos^2 x\,dx = \int \frac{\sin^2 2x}{4}\,dx$$

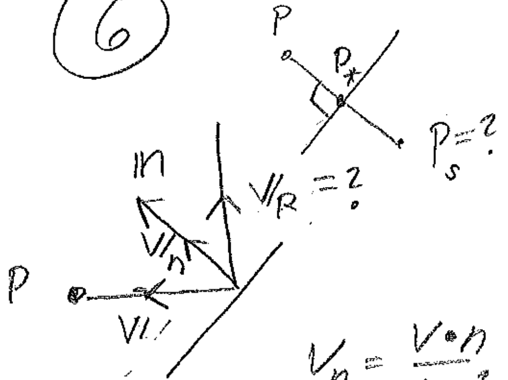
$$= \frac{1}{8}\int (1-\cos 4x)\,dx = \frac{1}{8}\left(x - \frac{\sin 4x}{4}\right) \Big|_0^{\pi/2} = \frac{\pi}{16}$$

b

$$\int \frac{x}{x^2+x^{-2}}\,dx = \int \frac{x^3}{x^4+1}\,dx = \left\{\begin{aligned} t &= x^4 \\ dt &= 4x^3 dx \end{aligned}\right\} = \frac{1}{4}\int \frac{dt}{t+1} =$$

$$= \frac{1}{4}\ln|t+1| = \frac{1}{4}\ln|x^4+1| \Big|_0^1 = \frac{1}{4}\ln 2$$

6



$$P_* - P = P_s - P_* \qquad P_s = 2P_* - P$$

$$P_*: \begin{cases} x = 2+3t \\ y = -t \\ z = -3+4t \end{cases} \qquad 3(2+3t) - (-t) + 4(-3+4t) = 1$$

$$t = \frac{7}{26} \qquad P_s = \left(\frac{47}{13}, -\frac{7}{13}, -\frac{11}{13}\right)$$

$$V_n = \frac{V\cdot n}{|n|^2}n \qquad V_R = V_n + V_n - V = 2V_n - V \qquad V_R = \left(\frac{58}{13}, -\frac{2}{13}, \frac{47}{13}\right)$$

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$$V = \frac{\pi R^2 H}{3} \qquad A = \pi R^2 + \pi R\sqrt{R^2 + H^2}$$

$$A(R) = \pi R^2 + \pi R\sqrt{R^2 + \left(\frac{3V}{\pi R^2}\right)^2}$$

$$A'(R) = 2\pi R + \pi\sqrt{} + \pi R \frac{1}{2\sqrt{}}\cdot(2R + \ldots)$$

$$A'(R) = 0 \iff R = \sqrt[3]{\frac{3V}{\pi}} \Big/ \sqrt{2} \qquad H = 2\sqrt[3]{\frac{3V}{\pi}}$$