MVE340 26/8-2016 SVAR

① ⓐ $\frac{x^2(1-\frac{1}{x^2})}{x^3(1+\frac{1}{x^2})} = \underbrace{\frac{1}{x}}_{\to 0}\,\underbrace{\frac{1-x^{-2}}{1+x^{-2}}}_{\to 1} \to 0,\ x\to\infty$

ⓑ $f'(x) = 10x+4 \quad f'(-1) = -6$ tgt $y = -2-6(x+1) = -6x-8$

nom $y = -2+\frac{1}{6}(x+1) = \frac{x-11}{6}$

ⓒ $\frac{x^3-x}{x-1} = \frac{x(x^2-1)}{x-1} = x(x+1)$

$$\begin{array}{ccccc} & -1 & & 0 & \\ + & & - & & + \end{array}$$

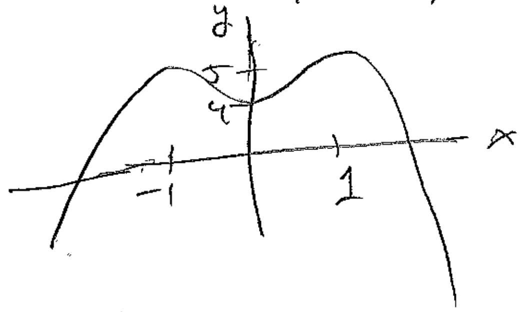
$-1 \le x \le 0$

ⓓ $f(-1.7) \approx f(-2) + \frac{f(-2)-f(-1)}{-2-(-1)}(-1.7-(-1)) = 1.5 + \frac{1.5-2.3}{-1}(-0.7)$

$= 1.5 + 0.56 = 2.06$

②a $f' = -4x^3+4x = 4x(1-x^2) = 0 \Leftrightarrow x_1 = 0\ x_{2,3} = \pm 1$

$$\begin{array}{c|ccccccc} & & -1 & & 0 & & 1 & \\ f' & + & & - & & + & & - \\ f & \nearrow & 5 & \searrow & 4 & \nearrow & 5 & \searrow \end{array}$$



②b

x	y
0	-6
1	-1
2	28

$x_0 = 2$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.078 \quad |f(x_1)| = 0.07$

$x_2 = 1.072 \quad |f(x_2)| < 3\cdot 10^{-4}$

②c $x + x^2 = 2x+12 \Leftrightarrow x = -3$ eller $x = 4$

$\int_{-3}^{4}(12+x-x^2)\,dx = 343/6$

③a $2A + 3(2Ax+B) - 4Ax^2 - 4Bx = \underbrace{2A+3B}_{10} + \underbrace{(6A-4B)}_{-9}x\ \underbrace{-4A}_{-2}x^2$

$A = \frac{1}{2} \quad B = 3$

③b $y = Ce^{-5t} \qquad C - 5C = 2 \quad C = -\frac{1}{2}$

3c $r^2+4r+13=0 \quad r_{1,2}=-2\pm 3i$

$y=e^{-2t}(C_1\cos 3t + C_2 \sin 3t) \quad C_1 = 2$

$y' = -2e^{-2t}(\quad) + e^{-2t}(-3C_1 \sin(\,) + 3C_2\cos(\,))$

$-2C_1+3C_2 = -3 \quad C_2 = 1$

4a $-1+4t-2(2-3t)-3(0+2t)=-9 \quad 4t=-4 \quad t=-1$

$(-5,5,-2)$

4b
```
>> A=[1 0 -1; 4 3 0; -1 6 4]
>> b=[1;5;8]
>> x = A\b
```

4c $\begin{cases} 55k+15m=-21.4 \\ 15m+5k=-1.5 \end{cases} \quad \begin{matrix} k=-1.69 \\ m=4.77 \end{matrix}$

4d
```
>> A=[1 1;2 1;3 1;4 1;5 1]
>> y=[3.3;1.4;-0.7;-2.1;-3.4]
>> A\y
```

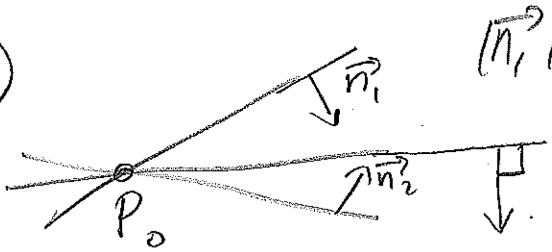
5 $\int_0^1 (1-t^2)(1+t^2)^2 dt = \int_0^1 (1-t^4)(1+t^2)\,dt$

$= 1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}$

$\int_0^1 \frac{x}{1+x^4}dx + \int_0^1 \frac{x^3}{1+x^4}dx = \left[\frac{1}{2}\arctan x^2 + \frac{1}{4}\ln(1+x^4)\right]_0^1$

$= \frac{\pi}{8}+\frac{1}{4}\ln 2$

(6)

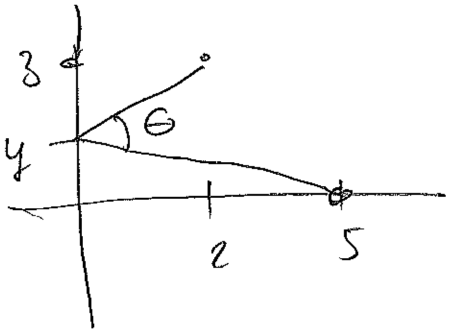


$|\vec{n_1}| = |\vec{n_2}| = 1 \quad \vec{n_1}\cdot\vec{n_2} \le 0$

$\vec{n_1} - \vec{n_2}$ är normal

$$P_0: \begin{cases} 2x - y + 5z = 1 \\ 3x - 2y + z = 2 \end{cases}$$

(7)



$$\theta = \pi - \arctan\frac{2}{3-y} - \arctan\frac{5}{y}$$

$$\theta' = \frac{-1}{1+\left(\frac{2}{3-y}\right)^2}\cdot\frac{2}{(3-y)^2} + \frac{1}{1+\left(\frac{5}{y}\right)^2}\cdot\frac{5}{y^2}$$

$\theta' = 0 \Leftrightarrow y = 5 - 2\sqrt{5}$