MVE365
Matematik Chalmers
Tentamensskrivning i Ämnesdidaktisk problemlösning, MPLOL
Datum: 2013-03-14, 14:00-18:00
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Hjälpmedel: Inga.
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Del 1: Geometry

1. Show that the area of a regular octagon is equal to the product of the longest and the shortest of its diagonals. (6p)

2. Construct a triangle given the angle at one of its vertices and the two segments into which the angle bisector of this angle divides the opposite side. (6p)

3. The altitudes from B and from C in $\triangle ABC$ intersect and the point of intersection divides each of them in the ratio $2:1$, counted from the vertex. Show that $\triangle ABC$ is equilateral. (6p)

4.(a) Prove that the three altitudes of a triangle (regarded as straight lines) intersect at one point. When do the three segments we call altitudes intersect? (6p)

(b) One of the formulas below gives the length of the median from C in a triangle, expressed in the lengths of its sides. Which one? You will get points for each formula you eliminate. Give reasons for your statements! (max 6p)

$$\textbf{(a)}\ m_c^2 = \frac{2(a^2+b^2)-c^2}{4}; \qquad \textbf{(b)}\ m_c^2 = \frac{(a^2+b^2+c^2)-2(c-a)^2}{4};$$
$$\textbf{(c)}\ m_c^2 = \frac{2c^2-(a^2+b^2)}{4}; \qquad \textbf{(d)}\ m_c^2 = \frac{5c^2-(a^2+b^2)}{4}.$$

Trigonometry, vectors, coordinate geometry and complex numbers may not be used.

Del 2: Strategies and methods

5. Read the problem and its solution carefully and answer the questions below. The problem can give a maximum of 4p.

Find all positive integers n such that the number n^4-3n^2+9 is a prime.

Solution. We have

$$n^4-3n^2+9=(n^2+3)^2-9n^2=(n^2-3n+3)(n^2+3n+3).$$

For this product to be a prime, one of the factors has to be 1 or -1. It is easily seen that none of them can be -1. The solutions of $n^2 - 3n + 3 = 1$ and $n^2 + 3n + 3 = 1$ are $n = \pm 1; \pm 2$. All these n-values give primes (7 and 13, respectively).

Questions: (1) What is the idea/strategy behind the solution?
(2) Can you vary the problem so that you keep the idea, but get a technically easier solution?

6. Read the solution of example 6.2 from chapter 6 of *Problem-Solving Strategies for Efficient and Elegant Solutions*, by Posamentier and Krulik (enclosed find a copy of the relevant pages).
(1) What is the strategy used?
(2) Do you consider the problem to be solved?
(3) If yes, what is the crucial observation; if no, where in the proof is there a flaw?
The problem can give a maximum of 4p.

7. *ANALOGY:* A tetrahedron can be seen on the one hand as the figure in three dimensions which is analogous to a triangle, on the other hand as a quadrilateral with four vertices which do not lie in one plane. Below you can read the formulation of a problem, treated in the course. Formulate and prove the corresponding statement for a tetrahedron.
The problem can give a maximum of 8p.

Given a quadrilateral, show that the segments connecting the midpoints of opposite sides and the segment connecting the midpoints of the diagonals pass through one point.

8. In the example treated in problem 6 you are supposed to show that a segment is longer than another segment. List some ways to show such an inequality. (max 4p)

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