Anonym kod	MVE415 Matematisk analys, del 1 170816	Sidnr 1	Poäng

1. Till nedanstående uppgifter skall lösningar redovisas, samt svar anges, på anvisad plats (endast lösningar och svar på detta blad, och på anvisad plats, beaktas).

(a) Bestäm med hjälp av derivatans definition $f'(x)$ då $f(x) = \dfrac{1}{1+x^2}$. (2p)

Lösning:

$$\frac{\frac{1}{1+x^2}-\frac{1}{1+a^2}}{x-a} = \frac{\overbrace{(a-x)}^{-\cancel{(x-a)}}(a+x)}{\cancel{(x-a)}(1+x^2)(1+a^2)} \to \frac{-2a}{(1+a^2)^2},\ x\to a$$

Svar: $-2a/(1+a^2)^2 = f'(a)$

(b) Bestäm lokala max/min till funktionen $f(x) = x^3 + \dfrac{3}{x}$. (3p)

Lösning:

$$f'(x) = 3x^2 - \frac{3}{x^2} = \frac{3(x^4-1)}{x^2} \qquad f' = 0 \Leftrightarrow x = \pm 1$$

-1	0	1	x

$+$	$-$	$-$	$+$
$\nearrow$	$\searrow$	$\searrow$	$\nearrow$

Svar: $x=-1$ lok max $\quad x=1$ lok min

(c) Lös ekvationen $|2x+3| - 4 = 5x$. (3p)

Lösning:

$x \geq -\frac{3}{2}$: $2x+3-4 = 5x \Leftrightarrow -1 = 3x \Leftrightarrow x = -\frac{1}{3}$ ok

$x < -\frac{3}{2}$: $-2x-3-4 = 5x \Leftrightarrow -7 = 7x \Leftrightarrow x = 1$ nej

Svar: $x = -1/3$

Var god vänd!

(d) Ange den primitiva funktion till $f(x) = \frac{3}{x^2} - \frac{6}{x^4}$ som uppfyller $F(1) = 5$. (2p)

Lösning:

$$F(x) = \frac{3x^{-2+1}}{-1} - \frac{6x^{-4+1}}{-3} + C$$

$$= -\frac{3}{x} + \frac{2}{x^3} + C$$

$$F(1) = -3 + 2 + C = 5 \iff C = 6$$

Svar: $F(x) = -\frac{3}{x} + \frac{2}{x^3} + 6$

(e) Bestäm inversen till funktionen $y(x) = \left(\frac{2}{x+1} + 1\right) / \left(\frac{3}{x+1} + 2\right)$. (3p)

Lösning:

$$y = \frac{x+3}{2x+5} \iff 2xy + 5y = x + 3$$

$$\iff 2xy - x = 3 - 5y$$

$$\iff x(2y-1) = 3 - 5y$$

$$\iff x = \frac{3-5y}{2y-1} = f^{-1}(y)$$

Svar: ...

(f) Bestäm ekvationen för tangenten till f's graf i punkten där $x = \pi$ för funktionen $f(x) = \sin(x/2) + x\cos(x/2)$. Ange också tangentens skärningspunkt med x-axeln. (3p)

Lösning:

$$f'(x) = \frac{1}{2}\cos\frac{x}{2} + \cos\frac{x}{2} - \frac{x}{2}\sin\frac{x}{2}$$

$$f'(\pi) = -\frac{\pi}{2} \qquad \text{tgt: } y = 1 - \frac{\pi}{2}(x - \pi)$$

skärning: $y = 0 \iff \frac{\pi}{2}(x-\pi) = 1 \quad x = \pi + \frac{2}{\pi}$

Svar: Tgt: $y = 1 - \frac{\pi}{2}(x-\pi)$ skärn: $x = \pi + \frac{2}{\pi}$

2 $f'(x) = \dfrac{3x^2(x^2-3) - x^3 \cdot 2x}{(x^2-3)^2} = \dfrac{(x^2-9)x^2}{(x^2-3)^2}$

$f'(x) = 0 \Leftrightarrow x = \pm 3$
$x = 0$

x	-3		$-\sqrt{3}$		0		$\sqrt{3}$		3	
$+$	0	$-$		$-$	0	$-$		$-$	0	$+$
$\nearrow$		$\searrow$		$\searrow$		$\searrow$		$\searrow$		$\nearrow$

$\lim\limits_{x \to \sqrt{3}^{\pm}} = \pm\infty \qquad \lim\limits_{x \to -\sqrt{3}^{\pm}} = \mp\infty$

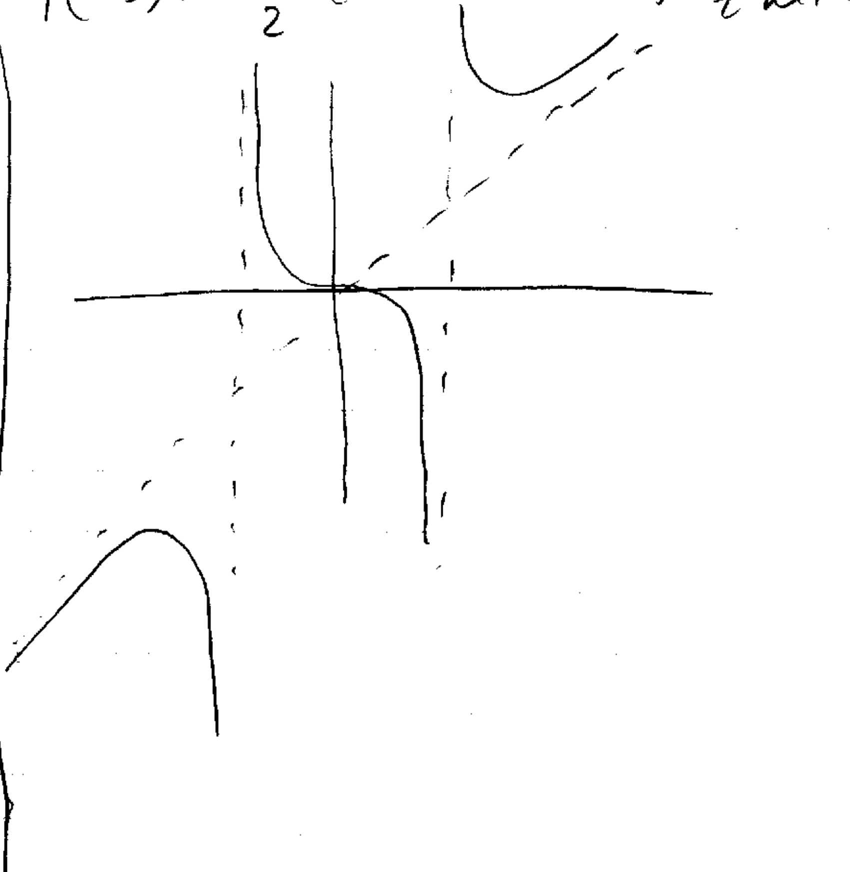
x	y
-3	$-\frac{9}{2}$
0	0
3	$\frac{9}{2}$

$\dfrac{y}{x} = \dfrac{x^3}{x(x^2-3)} = \dfrac{x^3}{x^3(1-3/x^2)} \to 1$

$y - 1 \cdot x = \dfrac{3x}{x^2-3} \to 0$

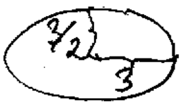
asymptoter : $y = x \quad x = \pm\sqrt{3}$

lok max/min : $f(-3) = -\frac{9}{2}$ lok max $f(3) = \frac{9}{2}$ lok min



3a $(x+2)^2 + 4(y-1)^2 = 9$

$(-2, 1)$



störst: 3

minst: $\frac{3}{2}$

3b $y = 0 \quad x = -2 \pm \sqrt{5}$

$2x + 4 + 8yy' - 8y' = 0$

$y' = -\dfrac{2x+4}{8y-8} = \dfrac{\pm\sqrt{5}}{4}$

4 $\lim\limits_{x \to -2} \dfrac{1}{x^2+2x} + \dfrac{1}{4+2x} = \lim\limits_{x \to -2} \dfrac{\cancel{2+x}}{2\cancel{(x+2)}x} = -\dfrac{1}{4}$

5 $y = e^{-x}(C_1\cos 2x + C_2\sin 2x)$

$y' = e^{-x}(-C_1\cos 2x - C_2\sin 2x - 2C_1\sin 2x + 2C_2\cos 2x)$

$y'' = e^{-x}(C_1\cos 2x + C_2\sin 2x + 2C_1\sin 2x - 2C_2\cos 2x$

$+ 2C_1\sin 2x - 2C_2\cos 2x - 4C_1\cos 2x - 4C_2\sin 2x)$

$y'' - 2y' + y = e^{-x}(C_1 + 2C_1 - 4C_2 + C_1 - 2C_2 - 2C_2 - 4C_1)\cos 2x$

$+ e^{-x}(C_2 + 2C_2 + 4C_1 + C_2 + 2C_1 + 2C_1 - 4C_2)\sin 2x$

$= e^{-x}\underbrace{(C_1 - 4C_2)}_{=4}\cos 2x + e^{-x}\underbrace{(8C_1)}_{=0}\sin 2x \qquad C_2 = -1$

6 $x^2 + 6x + 9 = 1 - 6x + 9x^2 \Leftrightarrow -2 - 3x + 2x^2 = 0$

$\Leftrightarrow x = \dfrac{3 \pm \sqrt{9 - 4\cdot(-2)\cdot 2}}{4} = \dfrac{3 \pm 5}{4} = \begin{cases} 2 \\ -1/2 \end{cases} \leftarrow$ test

7 a $f'(x) = e^x - 3e^{-2x} \quad f(x) = e^x + \frac{3}{2}e^{-2x} + C$

$C = 1/2 - 3/2 - 1 = -2$

7b $f'(x) = \dfrac{e^{3x} - 3}{e^{2x}} \quad x = \dfrac{\ln 3}{3} \quad f'(0) < 0, \; f'(1) > 0$ lok min

8 i $\dfrac{1 - \cos 2x}{2} \cdot \sin^2 2x = \dfrac{1 - \cos 4x}{4} - \dfrac{1}{2}\cos 2x \sin^2 2x$

$= D\left(\dfrac{x}{4} - \dfrac{\sin 4x}{16} - \dfrac{1}{12}\sin^3 2x + C\right)$

8 ii $\dfrac{e^x - e^{-x}}{(e^x + e^{-x})^2} = \{t = e^x + e^{-x}\} = \dfrac{e^x - e^{-x}}{t^2} \cdot \dfrac{dx}{dt}\dfrac{dt}{dx}$

$= \left\{\dfrac{dx}{dt} = 1 / \dfrac{dt}{dx}\right\} = \dfrac{\cancel{e^x - e^{-x}}}{t^2} \cdot \dfrac{1}{\cancel{e^x - e^{-x}}} \dfrac{dt}{dx}$

$= -\dfrac{d}{dt}\dfrac{1}{t} \cdot \dfrac{dt}{dx} = -\dfrac{d}{dx}\left(\dfrac{1}{t}\right) = \dfrac{d}{dx}\left(-\dfrac{1}{e^x + e^{-x}} + C\right)$

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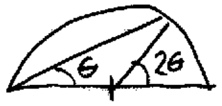

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