Problem 3.1/

areasatsen: $A = \frac{1}{2} ab \sin\gamma$ (*)

cosinussatsen: $c^2 = a^2 + b^2 - 2ab\cos\gamma \Rightarrow \cos\gamma = \frac{a^2+b^2-c^2}{2ab}$

trig: ettan: $\sin^2\gamma + \cos^2\gamma = 1 \Rightarrow \sin^2\gamma = 1 - \cos^2\gamma$

konjugatregeln: $\sin^2\gamma = (1-\cos\gamma)(1+\cos\gamma) =$

$$= \left(1 - \frac{a^2+b^2-c^2}{2ab}\right)\left(1 + \frac{a^2+b^2-c^2}{2ab}\right)$$

$$= \frac{2ab - a^2 - b^2 + c^2}{2ab} \cdot \frac{2ab + a^2 + b^2 - c^2}{2ab}$$

$$= \frac{c^2 - (a^2 - 2ab + b^2)}{2ab} \cdot \frac{(a^2 + 2ab + b^2) - c^2}{2ab}$$

kvadreringsreglerna: $= \frac{c^2 - (a-b)^2}{2ab} \cdot \frac{(a+b)^2 - c^2}{2ab}$

konjugatregeln: $= \frac{(c+(a-b))(c-(a-b))}{2ab} \cdot \frac{((a+b)+c)((a+b)-c)}{2ab}$

$$= \frac{(c+a-b)(c+b-a)(a+b+c)(a+b-c)}{(2ab)^2}$$

Således:

(*): $A = \frac{1}{2} \cancel{a \cdot b} \cdot \sqrt{\frac{(c+a-b)(c+b-a)(a+b+c)(a+b-c)}{(2\cancel{ab})^2}}$

$$= \frac{1}{4} \cdot \sqrt{(c+a-b)(c+b-a)(a+b+c)(a+b-c)}$$

V. S. B.

Problem 3.2

$$\frac{\sin(u+v)+\sin(u-v)}{2} = \frac{\sin u \cos v + \cancel{\sin v \cos u} + su\,cv - \cancel{sv\,cu}}{2}$$

$$= \frac{2\,su\,cv}{2} = su\,cv \qquad (*)$$

Sätt $\left.\begin{array}{l} u+v=\alpha \\ u-v=\beta \end{array}\right\} \Longleftrightarrow \left\{\begin{array}{l} u=\frac{\alpha+\beta}{2} \\ v=\frac{\alpha-\beta}{2} \end{array}\right.$

Då följer det från $(*)$ att: $\frac{\sin\alpha+\sin\beta}{2} = \sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$$\Longleftrightarrow \sin\alpha+\sin\beta = 2\cdot\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha-\sin\beta = \sin\alpha+\sin(-\beta) = 2\cdot\sin\frac{\alpha+(-\beta)}{2}\cdot\cos\frac{\alpha-(-\beta)}{2}$$
$$= 2\cdot\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\frac{\cos(u+v)+\cos(u-v)}{2} = \frac{cu\,cv - \cancel{su}\,sv + cu\,cv + \cancel{su}\,sv}{2}$$

$$= \frac{2\,cu\,cv}{2} = cu\,cv$$

$$\left.\begin{array}{l} u+v=\alpha \\ u-v=\beta \end{array}\right\} \Rightarrow \cos\alpha+\cos\beta = 2\cdot\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\frac{\cos(u+v)-\cos(u-v)}{2} = \frac{\cancel{cu}\,cv - su\,sv - (\cancel{cu}\,cv + su\,sv)}{2}$$

$$= \frac{-2\,su\,sv}{2} = -su\,sv$$

$$\left.\begin{array}{l} u+v=\alpha \\ u-v=\cancel{\alpha}\beta \end{array}\right\} \Rightarrow \cos\alpha-\cos\beta = -2\cdot\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$$

Problem 3.3

$$\tan 2v = 4\tan v$$

$$\Leftrightarrow \frac{2\tan v}{1-\tan^2 v} = 4\tan v$$

$$\Leftrightarrow 0 = 4\tan v - \frac{2\tan v}{1-\tan^2 v}$$

$$\Leftrightarrow 0 = 2\cdot\tan v\cdot\left(2-\frac{1}{1-\tan^2 v}\right)$$

$$\Leftrightarrow 0 = 2\cdot\tan v\cdot\frac{1-2\tan^2 v}{1-\tan^2 v}$$

$\Leftrightarrow$ $\tan v = 0$ eller $1-2\tan^2 v = 0$

$v = 0 + k\pi$

$v = k\pi$, där $k \in \mathbb{Z}$

$1 = 2\tan^2 v$

$\frac{1}{2} = \tan^2 v$

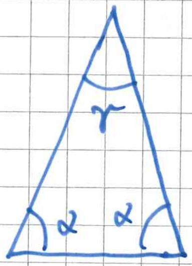
$\pm\sqrt{\frac{1}{2}} = \tan v$

$v = \arctan\left(\pm\frac{1}{\sqrt{2}}\right) + k\pi$

$v = \pm\arctan\frac{1}{\sqrt{2}} + k\pi$

där $k \in \mathbb{Z}$

Problem 3.4



$\gamma = 180° - 2\alpha$

$\tan\gamma = 2\sin\alpha$

$\tan(180° - 2\alpha) = 2\sin\alpha$

$-\tan(2\alpha) = 2\sin\alpha$

$\frac{-\sin 2\alpha}{\cos 2\alpha} = 2\sin\alpha$

$\frac{-2\sin\alpha\cos\alpha}{\cos^2\alpha-\sin^2\alpha} = 2\sin\alpha$

$-\cos\alpha = \cos 2\alpha \Leftrightarrow \cos(180° - \alpha) = \cos 2\alpha \Leftrightarrow$ (*)

(*) $180° - \alpha = 2\alpha$

$180° = 3\alpha$

$60° = \alpha$

$\gamma = 60°$

Problem 3.5/:

$$\tan(u+v) = \frac{\tan u + \tan v}{1-\tan u \tan v} = \frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\cdot\frac{3}{4}} = \frac{\frac{4+21}{28}}{1-\frac{3}{28}} =$$

$$= \frac{\frac{25}{28}}{\frac{25}{28}} = 1$$

$$\tan(u+v) = 1 \implies u+v = \arctan 1 + k\pi$$

$$= \frac{\pi}{4} + k\pi, \text{ dān } k \in \mathbb{Z}.$$

Problem 3.6/

$$\gamma = 180° - (\alpha+\beta)$$

$$\sin\alpha + \sin\beta + \sin\gamma = s\alpha + s\beta + s(180° - (\alpha+\beta))$$

$$= s\alpha + s\beta + s(\alpha+\beta)$$

$$= s\alpha + s\beta + s\alpha\, c\beta + s\beta\, c\alpha$$

$$= s\alpha(1+c\beta) + s\beta(1+c\alpha) = (*)$$

$$1 + c\beta = 1 + 2\cdot\cos^2\frac{\beta}{2} - 1 = 2\cdot\cos^2\frac{\beta}{2}$$

$$1 + c\alpha = \quad \dots \quad = 2\cos^2\frac{\alpha}{2}$$

$$s\alpha = 2\cdot s\frac{\alpha}{2}\, c\frac{\alpha}{2}$$

$$s\beta = 2\, s\frac{\beta}{2}\, c\frac{\beta}{2}$$

$$(*) = 2\cdot s\frac{\alpha}{2} c\frac{\alpha}{2}\cdot 2c^2\frac{\beta}{2} + 2\cdot s\frac{\beta}{2}\cdot c\frac{\beta}{2}\cdot 2\cdot c^2\frac{\alpha}{2}$$

$$= 4\cdot c\frac{\alpha}{2}\, c\frac{\beta}{2}\cdot\left(s\frac{\alpha}{2} c\frac{\beta}{2} + s\frac{\beta}{2} c\frac{\alpha}{2}\right)$$

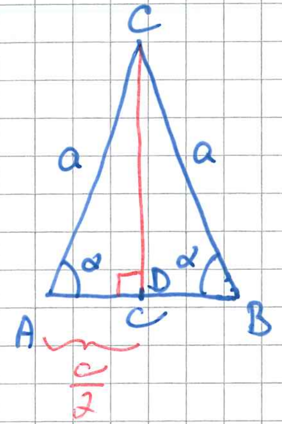
$$= 4\cdot c\frac{\alpha}{2}\, c\frac{\beta}{2}\cdot s\frac{\alpha+\beta}{2}$$

$$= 4\cdot c\frac{\alpha}{2}\, c\frac{\beta}{2}\cdot c\left(90° - \frac{\alpha+\beta}{2}\right)$$

$$= 4\cdot c\frac{\alpha}{2}\, c\frac{\beta}{2}\cdot c\left(\frac{180° - (\alpha+\beta)}{2}\right)$$

$$= 4\cdot\cos\frac{\alpha}{2}\cdot\cos\frac{\beta}{2}\cdot\cos\frac{\gamma}{2} \qquad \text{V.S.B.}$$

Problem 3.7



(a) ΔADC: $\cos\alpha = \dfrac{|AD|}{|AC|} = \dfrac{c/2}{a}$

$$\Rightarrow a\cdot\cos\alpha = \frac{c}{2}$$

$$\Rightarrow c = 2\cdot a\cdot\cos\alpha$$

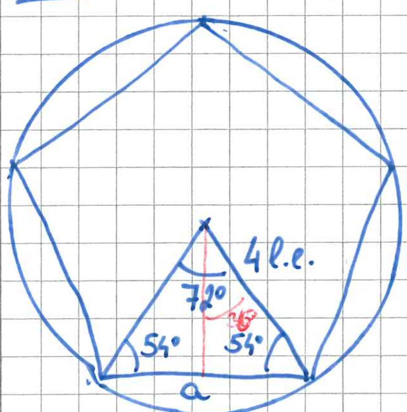
V. S. B.

(b) cosinussatsen i ΔABC:

$$c^2 = a^2 + a^2 - 2\cdot a\cdot a\cdot\cos\gamma$$
$$= 2a^2 - 2a^2\cos\gamma$$
$$= 2a^2(1-\cos\gamma)$$

V. S. B.

Problem 3.8



(a) $\sin 36° = \frac{a}{2} \div 4$

$\uparrow$ katet $\qquad \uparrow$ hypotenusan

$$\Downarrow$$

$$a = 8\cdot\sin 36° = 8\cdot\sqrt{\frac{5-\sqrt{5}}{8}}$$
$$= 8\cdot\sqrt{\frac{10-2\sqrt{5}}{16}} = \frac{8}{4}\cdot\sqrt{10-\sqrt{4\cdot 5}}$$
$$= 2\sqrt{10-\sqrt{20}}\ \text{l.e.}$$

omkretsen: $o = 5a = 10\sqrt{10-\sqrt{20}}$ l.e.

arean: $A = 5\cdot\underbrace{\tfrac{1}{2}\cdot 4\cdot 4\cdot\sin 72°}_{\text{areasatsen}} = 40\cdot\sin 72°$

$$= 10\sqrt{10+\sqrt{20}}\ \text{a.e.}$$

(b) $a = 8\cdot\sin\dfrac{360°}{2k} = 8\cdot\sin\dfrac{180°}{k}$ l.e.

$o = k\cdot a = 8k\sin\dfrac{180°}{k}$ l.e.

$A = k\cdot\frac{1}{2}\cdot 4\cdot 4\cdot\sin\dfrac{360°}{k} = 8k\sin\dfrac{360°}{k}$ a.e.