Lösningar dugga, 2018-12-17.

Uppgift 1. Integrerande faktor: $\mathrm{e}^{\int -2x\,dx} = \mathrm{e}^{-x^2}$

$$\mathrm{e}^{-x^2}y' - \mathrm{e}^{-x^2}2xy = \mathrm{e}^{-x^2}x\mathrm{e}^{x^2} = x \Leftrightarrow \left(\mathrm{e}^{-x^2}y\right)' = x \Leftrightarrow \mathrm{e}^{-x^2}y = \frac{x^2}{2} + C$$

$$y(x) = (\frac{x^2}{2} + C)\mathrm{e}^{x^2}$$

$$y(0) = 1 \Rightarrow C = 1$$

$$y(x) = (\frac{x^2}{2} + 1)\mathrm{e}^{x^2}$$

Uppgift 2. Homogenlösning till $y'' + 2y' + y = 0$: Karakt. ekv. $r^2 + 2r + 1 = 0$ med dubbelrot $r_{1,2} = -1$ ger

$$y_h(x) = (C_1 + C_2x)\mathrm{e}^{-x}$$

Partikulärlösning till $y'' + 2y' + y = \sin(x)$: Ansätt $y_p(x) = A\sin(x) + B\cos(x)$. Insättning i ekvationen ger $A = 0, B = -\frac{1}{2}$ och $y_p(x) = -\frac{1}{2}\cos(x)$. Vi får

$$y(x) = y_h(x) + y_p(x) = (C_1 + C_2x)\mathrm{e}^{-x} - \tfrac{1}{2}\cos(x)$$

Vi har

$$y'(x) = (-C_1 + C_2 - C_2x)\mathrm{e}^{-x} + \tfrac{1}{2}\sin(x)$$

Randvillkor

$$y(0) = C_1 - \tfrac{1}{2} = -1 \Rightarrow C_1 = -\tfrac{1}{2},\;\; y'(0) = -C_1 + C_2 = 1 \Rightarrow C_2 = \tfrac{1}{2}$$

$$y(x) = \tfrac{1}{2}(x-1)\mathrm{e}^{-x} - \tfrac{1}{2}\cos(x)$$

Uppgift 3. Vi har $f(x) = 2\sqrt{x}$ med $f'(x) = \frac{1}{\sqrt{x}}$ och får

$$S = \int_1^2 2\pi f(x)\sqrt{1 + f'(x)^2}\,d = \int_1^2 2\pi\, 2\sqrt{x}\sqrt{1 + \left(\frac{1}{\sqrt{x}}\right)^2}\,dx = 4\pi\int_1^2 \sqrt{x+1}\,dx =$$

$$= 4\pi\,\tfrac{2}{3}\left[(x+1)^{3/2}\right]_1^2 = \frac{8\pi}{3}\left(3^{3/2} - 2^{3/2}\right) = \frac{8\pi}{3}(3\sqrt{3} - 2\sqrt{2})$$