uppg 1

a) $\nabla f = (2y-3x^2,\ 2y+2x)$ , $\nabla f(2,1) = (-10,6)$

$D_u f(2,1) = \nabla f(2,1) \bullet u = (-10,6)\bullet(\frac{3}{5},\frac{4}{5}) = -6+\frac{24}{5} = \underline{\underline{\frac{-6}{5}}}$

b) $\underline{\underline{5f_1 + 3f_3}}$

c) $f = \ln(x+2y)$ , $f(1,0)=0$ | $f_{11} = -1/(x+2y)^2$ , $f_{11}(1,0) = -1$

$f_1 = 1/(x+2y)$ , $f_1(1,0) = 1$ | $f_{12} = -2/(x+2y)^2$ , $f_{12}(1,0) = -2$

$f_2 = 2/(x+2y)$ , $f_2(1,0) = 2$ | $f_{22} = -4/(x+2y)^2$ , $f_{22}(1,0) = -4$

$$P_2(x,y) = \underline{\underline{x-1+2y-\frac{1}{2}\left((x-1)^2+4(x-1)y+4y^2\right)}}$$

$$f(1.1,\,0.2) \approx P_2(1.1,\,0.2) = 0.1+0.4-\frac{1}{2}(0.01+0.08+0.16) = \\ = \underline{\underline{0.375}}$$

d) $\mathbb{r}(2,1) = (4,3,1)$

$$\mathbb{r}_u' \times \mathbb{r}_v' = \begin{vmatrix} \mathbb{i} & \mathbb{j} & \mathbb{k} \\ 2u & 1 & v^3 \\ 0 & 2v & 3uv^2 \end{vmatrix} = (3uv^2-2v^4,\ -6u^2v^2,\ 4uv)$$

$(\mathbb{r}_u' \times \mathbb{r}_v')(2,1) = (4,-24,8) \parallel (1,-6,2)$

Matematisk symbol som betyder "parallell med"

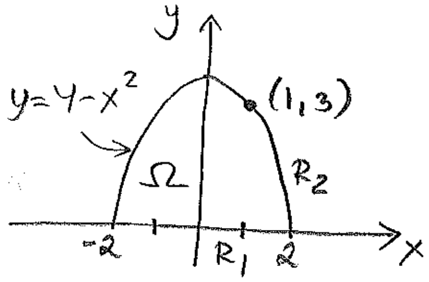
Svar: t.ex. $\mathbb{N} = \underline{\underline{(1,-6,2)}}$

Uppg 2 a) $\begin{cases} f_1 = y-3=0 \\ f_2 = x-1=0 \end{cases} \Leftrightarrow (x,y)=(1,3)$

$$\mathcal{H}(x,y)=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \det \mathcal{H}(1,3) = -1 < 0$$

så $(1,3)$ är den enda kritiska punkten och det är en sadelpunkt.

b)




(I) Kritiska punkter saknas i det inre av $\Omega$

(II) Sing. punkter saknas.

(III) $R_1: y=0: \; f(x,0)=-3x, \; f(2,0)=-6, \; f(-2,0)=6$

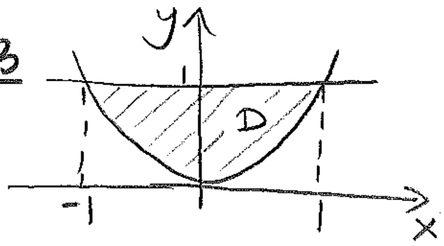
$R_2: y=4-x^2: \; f(x,4-x^2)=x(4-x^2)-3x-(4-x^2)=$
$= -x^3+x^2+x-4 = g(x)$

$$g'(x)=-3x^2+2x+1=0 \Leftrightarrow$$
$$x^2-\frac{2}{3}x-\frac{1}{3}=0 \Leftrightarrow$$
$$x=\frac{1}{3}\pm\sqrt{\frac{1}{9}+\frac{1}{3}}=\frac{1\pm 2}{3} \Leftrightarrow$$
$$x=\frac{-1}{3} \text{ eller } x=1$$

$$g\left(\frac{-1}{3}\right)=\frac{1}{27}+\frac{1}{9}-\frac{1}{3}-4=\frac{-113}{27}>-6$$
$$g(1)=-3$$

Svar: Största värde är 6 och minsta $-6$

Uppg 3




$$\int_{-1}^{1}\left(\int_{x^2}^{1} y\,dy\right)dx = \int_{-1}^{1}\left[\frac{y^2}{2}\right]_{x^2}^{1} dx =$$

$$= \frac{1}{2}\int_{-1}^{1}(1-x^4)\,dx = \frac{1}{2}\left[x-\frac{x^5}{5}\right]_{-1}^{1} = \underline{\underline{\frac{4}{5}}}$$

$$\int_{0}^{1}\left(\int_{-\sqrt{y}}^{\sqrt{y}} y\,dx\right)dy = \int_{0}^{1} 2y^{3/2}\,dy = 2\left[\frac{y^{5/2}}{5/2}\right]_{0}^{1} = \underline{\underline{\frac{4}{5}}}$$

Uppg 4

a)
$$\begin{cases} x = \rho\sin\phi\cos\theta \\ y = \rho\sin\phi\sin\theta \\ z = \rho\cos\phi \end{cases}$$

$(-1,0,0)$ motsvarar $(1, \frac{\pi}{2}, \pi)$ i de sfäriska koordinaterna

b)
$$\iiint_K z\,dV = \int_0^{2\pi}\int_0^{\pi/2}\int_0^{2} \rho\cos\phi\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta =$$

$$= 2\pi\left[\frac{\sin^2\phi}{2}\right]_0^{\pi/2}\left[\frac{\rho^4}{4}\right]_0^{2} = \underline{\underline{4\pi}}$$

alt.
$$\iiint_K z\,dV = \int_0^2 z\left(\iint_{x^2+y^2\le 4-z^2} dx\,dy\right)dz =$$

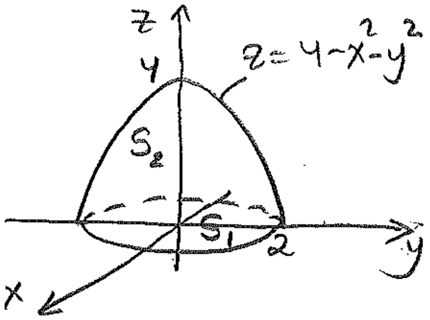
$$= \int_0^2 z\pi(4-z^2)\,dz = \pi\left[2z^2-\frac{z^4}{4}\right]_0^2 = \underline{\underline{4\pi}}$$

alt.
$$\iiint_K z\,dV = \iint_{x^2+y^2\le 4}\left(\int_0^{\sqrt{4-x^2-y^2}} z\,dz\right)dx\,dy =$$

$$= \iint_{x^2+y^2\le 4}\frac{1}{2}(4-x^2-y^2)\,dx\,dy = \frac{1}{2}\int_0^{2\pi}\left(\int_0^2 (4-r^2)r\,dr\right)d\theta =$$

$$= \pi\left[2r^2-\frac{r^4}{4}\right]_0^2 = \underline{\underline{4\pi}}$$

## uppg 5




$$\mathbb{F} = x\,\mathbf{i} + y\,\mathbf{j} + (z-1)\,\mathbf{k}$$

$$\iint_{S_1} \underbrace{\mathbb{F}}_{x\mathbf{i}+y\mathbf{j}-\mathbf{k} \text{ på } S_1} \cdot \underbrace{\hat{\mathbb{N}}\,dS}_{-\mathbf{k}\,dxdy \text{ på } S_1} = \iint_{x^2+y^2\le 4} dxdy = \underline{\underline{4\pi}}$$

$$\iint_{S_2} \underbrace{\mathbb{F}}_{x\mathbf{i}+y\mathbf{j}+(3-x^2-y^2)\mathbf{k} \text{ på } S_2} \cdot \underbrace{\hat{\mathbb{N}}\,dS}_{(2x\mathbf{i}+2y\mathbf{j}+\mathbf{k})\,dxdy \text{ på } S_2} = \iint_{x^2+y^2\le 4} (2x^2+2y^2+3-x^2-y^2)\,dxdy$$

$$= \iint_{x^2+y^2} (3+x^2+y^2)\,dxdy = \int_0^{2\pi}\left(\int_0^2 (3+r^2)\,r\,dr\right)d\theta$$

$$= 2\pi\left[\frac{3r^2}{2} + \frac{r^4}{4}\right]_0^2 = \underline{\underline{20\pi}}$$

Totalt flöde: $4\pi + 20\pi = \underline{\underline{24\pi}}$

b) $$\iint_{S_1+S_2} \mathbb{F}\cdot\hat{\mathbb{N}}\,dS = \iiint_D \operatorname{div}\mathbb{F}\,dV =$$

$$= 3\iiint_D dV = 3\int_0^4\left(\iint_{x^2+y^2\le 4-z} dxdy\right)dz =$$

$$= 3\int_0^4 \pi(4-z)\,dz = 3\pi\left[4z - \frac{z^2}{2}\right]_0^4 = \underline{\underline{24\pi}}$$

Uppg 6 (a) $f(x,0)=0 \to 0$
$f(x,x)=\frac{1}{2} \to \frac{1}{2} \neq 0$ $\Big\} \Rightarrow$ gränsvärde saknas

(b) $|f(r\cos\theta, r\sin\theta)| \le r \to 0$ så

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^2+y^2} = \underline{\underline{0}}$$

Uppg 7 a) $\begin{cases} x = 2\cos t \\ y = 2\sin t \\ z = 3-2\cos t \end{cases}$, $0 \le t \le 2\pi$

$$\int_C \mathbb{F} \cdot d\mathbb{r} = \int_0^{2\pi} \big(-2\sin t(-2\sin t) + 4\cos^2 t \cdot 2\cos t + (3-2\cos t)^3 2\sin t\big)\, dt =$$

$$= 4\int_0^{2\pi} \sin^2 t\, dt = \underline{\underline{4\pi}}$$

b) $$\operatorname{curl} \mathbb{F} = \begin{vmatrix} \mathbb{i} & \mathbb{j} & \mathbb{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & x^2 & z^3 \end{vmatrix} = (2x+1)\mathbb{k}$$

$z = 3-x$

$\hat{N}\, dS = (\mathbb{i} + \mathbb{k})\, dx\, dy$

$$\int_C \mathbb{F} \cdot d\mathbb{r} = \iint_{x^2+y^2 \le 4} (2x+1)\, dx\, dy = \underline{\underline{4\pi}}$$

Uppg 8c $\mathbb{F} = \nabla(x^2+2y^2)$, $\frac{dx}{2x} = \frac{dy}{4y} \Leftrightarrow 2\ln|x| = \ln|y| + C$

$y = Cx^2$

Fältlinjen genom $(2,1)$ är $\underline{\underline{y = \frac{1}{4}x^2}}$