

Lösningar MVE 475, 180830

$$1. a) \left. \frac{d}{dx} \frac{1}{(f(x))^2 + 1} \right|_{x=4} = \left. \frac{-2f(x)f'(x)}{((f(x))^2 + 1)^2} \right|_{x=4}$$

$$= \frac{-2f(4)f'(4)}{((f(4))^2 + 1)^2} = \frac{-2 \cdot 1 \cdot (-4)}{(1^2 + 1)^2} = 2$$

$$b) \int \frac{1}{(x-1)(x^2+3x+2)} dx = \int \frac{1}{(x-1)(x+1)(x+2)} dx$$

$$= \int \left(\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2} \right) dx =$$

$$= \frac{1}{6} \int \frac{1}{x-1} dx = \frac{1}{2} \int \frac{1}{x+1} dx + \frac{1}{3} \int \frac{1}{x+2} dx =$$

$$= \frac{1}{6} \ln|x-1| - \frac{1}{2} \ln|x+1| + \frac{1}{3} \ln|x+2| + C$$

$$c) \lim_{x \rightarrow 0} \frac{e^{x^2} - 1}{\cos x - 1} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{2xe^{x^2}}{-\sin x} =$$

$$= -2 \lim_{x \rightarrow 0} \left(\frac{1}{\frac{\sin x}{x}} \cdot e^{x^2} \right) = -2 \cdot \frac{1}{1} \cdot e^0$$

$$= -2.$$

1. d) Om $f^{-1}(x) = y$ så $f(y) = x$
Som vi deriverar implicit om får

$$f'(y) \frac{dy}{dx} = 1 \quad \rightarrow \quad y' = \frac{1}{f'(y)}$$

så

$$(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}$$

Vi har att

$$f^{-1}(e^4) = 2 \quad \text{ty} \quad f(2) = e^4$$

och

$$f'(x) = 2xe^{x^2} \Rightarrow f'(2) = 4e^4$$

så

$$\begin{aligned} (f^{-1}(e^4))' &= \frac{1}{f'(f^{-1}(e^4))} = \frac{1}{f'(2)} \\ &= \frac{1}{4e^4} \end{aligned}$$

2 a) Se boken!

$$b) y' \cos x - y \sin x = \cos(xy)(y + xy')$$

Insättning av $(0,1)$ ger

$$y'(0) \cos 0 - 1 \sin 0 = \cos(0 \cdot 1)(1 + 0 \cdot y')$$

så

$$y'(0) = 1$$

Insättning ger

$$y - y(0) = y'(0)(x - 0)$$

$\Rightarrow$

$$y = x + 1$$

3 a) Se boken!

b) Medelvärdessatsen

$$f'(c) = \frac{f(4) - f(2)}{4 - 2} = \frac{3 - 1}{2} = 1$$

men $f'(x) > 1$ för alla x , så finns

Ingen funktion som uppfyller $f'(x) > 1$

för alla x .

$$4. \int_e^{\infty} \frac{1}{x(\ln x)^2} dx = \lim_{t \rightarrow \infty} \int_e^t \frac{1}{x(\ln x)^2} dx =$$

$$\left[\begin{array}{l|l} u = \ln x & x = e \text{ ger } u = 1 \\ du = \frac{1}{x} dx & x = t \text{ ger } u = \ln t \end{array} \right] =$$

$$= \lim_{t \rightarrow \infty} \int_1^{\ln t} \frac{1}{u^2} du = \lim_{t \rightarrow \infty} \left[-\frac{1}{u} \right]_1^{\ln t}$$

$$= \lim_{t \rightarrow \infty} \left(-\frac{1}{\ln t} + 1 \right) = 1.$$

5. def. mängd: $x \neq -1$.

$$\lim_{x \rightarrow -1^+} f(x) = \text{"arctan}(-\infty)" = -\frac{\pi}{2}$$

$$\lim_{x \rightarrow -1^-} f(x) = \text{"arctan } \infty" = +\frac{\pi}{2}$$

$$\lim_{x \rightarrow \infty} f(x) = \text{arctan}\left(\lim_{x \rightarrow \infty} \frac{x-1}{x+1}\right) = \text{arctan } 1 = \frac{\pi}{4}$$

$$\lim_{x \rightarrow -\infty} f(x) = \text{arctan}\left(\lim_{x \rightarrow -\infty} \frac{x-1}{x+1}\right) = \text{arctan } 1 = \frac{\pi}{4}$$

$\Rightarrow y = \frac{\pi}{4}$ vågrät asymptot då $x \rightarrow \pm \infty$

derivera

$$f'(x) = \frac{1}{1 + \left(\frac{x-1}{x+1}\right)^2} \cdot \left(\frac{(x+1) - (x-1)}{(x+1)^2}\right) = \frac{2}{(x+1)^2 + (x-1)^2} =$$

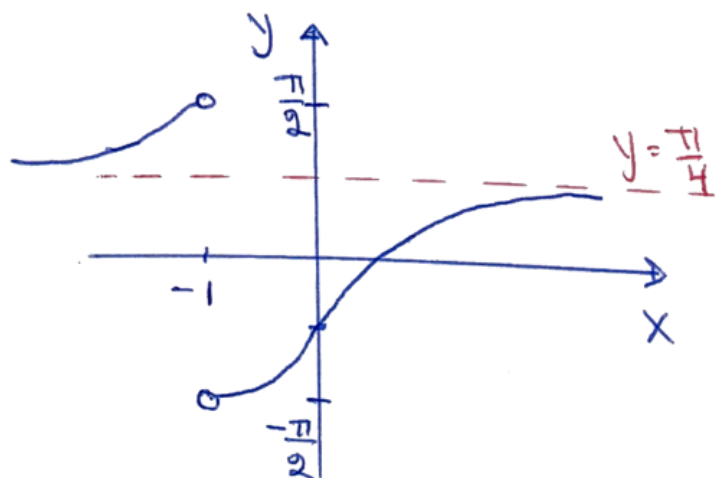
$$= \frac{1}{x^2 + 1} > 0 \Rightarrow f \text{ strängt växande}$$

på $(-\infty, -1)$ och $(-1, \infty)$

derivera $f'(x)$

$$f''(x) = -\frac{2x}{(x^2 + 1)^2}$$

x		-1		0	
f''	+	ej def	+		-
f	konvex		konvex	inf. pkt.	konkav



6. a) Falskt. $f(x) = -x^4$ har lokalt maximum i $x=0$ men $f''(0) = 0$.

b) Sant. $\int_0^1 \sin(x^2) dx \leq \int_0^1 x^2 dx = \frac{1}{3} < \frac{1}{2}$.

c) Sant. $F'(x) = \frac{d}{dx} \int_0^{x^2-2x} \frac{1}{1+t^4} dt =$
 $= \left[\begin{array}{l} u = x^2 - 2x \\ du = (2x-2) dx \end{array} \right] = \underbrace{\frac{d}{du} \int_0^u \frac{1}{1+t^4} dt \frac{du}{dx}}_{\text{kedjeregeln}} =$

$= \underbrace{\frac{1}{1+u^4}}_{\text{analysens huvudsats}} \frac{du}{dx} = \frac{1}{1+(x^2-2x)^4} \cdot (2x-2) =$

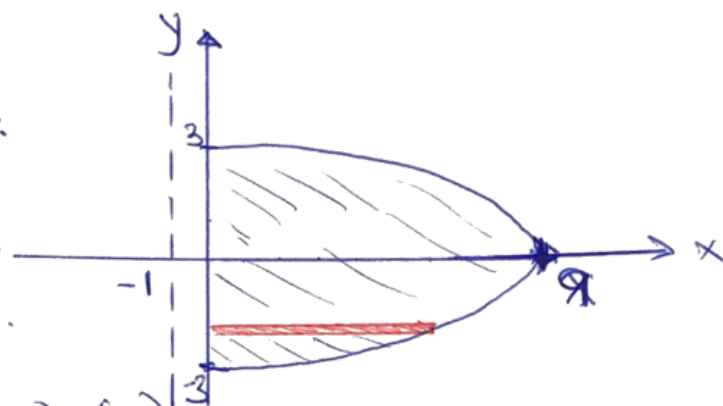
$= \frac{2(x-1)}{1+(x^2-2x)^4}$

x		1	
F'	-	0	+
F	↓		↑

lok. min. och även minsta värde.

7. Området:

Rotation kring
linjen $x = -1$.



Ytterradie: $(9 - y^2) - (-1)$

Innerradie: $0 - (-1)$

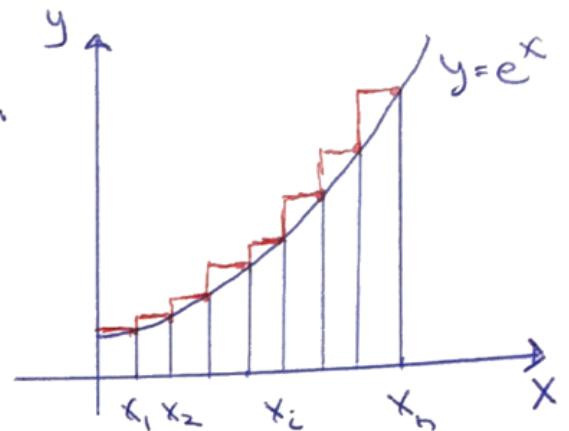
Rotationsvolymen blir då

$$\begin{aligned} V &= \pi \int_{-3}^3 \left((10 - y^2)^2 - 1^2 \right) dy = (\text{jämn}) = \\ &= 2\pi \int_0^3 (100 - 20y^2 + y^4 - 1) dy = \\ &= 2\pi \left[99y - \frac{20}{3}y^3 + \frac{y^5}{5} \right]_0^3 = \\ &= 2\pi \left[297 - 180 + \frac{243}{5} \right] = \\ &= 2\pi \left[\frac{117 \cdot 5 + 243}{5} \right] = \frac{2\pi}{5} \cdot (585 + 243) \\ &= \frac{1656\pi}{5} \quad \text{v.e.} \end{aligned}$$

8. $x_i^* = x_i$ höger ändpunkt.

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{n} = \frac{1}{n}$$

$$x_i = a + i \Delta x = \frac{i}{n}$$



Riemannsumman blir

$$\sum_{i=1}^n f(x_i^*) \Delta x = \sum_{i=1}^n e^{i/n} \cdot \frac{1}{n} = \sum_{i=1}^n e^{i/n} e^{-i/n} e^{i/n} \cdot \frac{1}{n} =$$

$$= \frac{e^{1/n}}{n} \underbrace{\sum_{i=1}^n \left(e^{\frac{1}{n}}\right)^{i-1}}_{\text{Geometrisk Summa}} = \frac{e^{1/n}}{n} \cdot \frac{1 - (e^{1/n})^n}{1 - e^{1/n}} =$$

$$= (e - 1) e^{1/n} \frac{1/n}{e^{1/n} - 1} = (e - 1) e^{1/n} \frac{1}{\frac{e^{1/n} - 1}{1/n}}$$

$$\rightarrow (e - 1) e^0 \cdot 1 = (e - 1) \quad \text{då } n \rightarrow \infty$$

Så

$$\int_0^1 e^x dx = e - 1$$