Formelblad MVE500, HT-2016

Trigonometri

$$\begin{aligned}
\cos(x+y) &= \cos x \cos y - \sin x \sin y \\
\sin(x+y) &= \sin x \cos y + \cos x \sin y \\
\cos x \cos y &= \frac{1}{2}\left(\cos(x-y) + \cos(x+y)\right) \\
\sin x \sin y &= \frac{1}{2}\left(\cos(x-y) - \cos(x+y)\right) \\
\sin x \cos y &= \frac{1}{2}\left(\sin(x-y) + \sin(x+y)\right) \\
\tan(x+y) &= \frac{\tan x + \tan y}{1 - \tan x \tan y}
\end{aligned}$$

Integraler

$$\int x^a\,dx = \frac{x^{a+1}}{a+1} + C, \quad a \neq -1 \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \frac{1}{\cos^2 x}\,dx = \tan x + C \qquad \int \frac{1}{\sin^2 x}\,dx = -\cot x + C$$

$$\int e^x\,dx = e^x + C \qquad \int a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C, \quad a \neq 0 \qquad \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

$$\int \frac{1}{\sqrt{a-x^2}}\,dx = \arcsin\frac{x}{\sqrt{a}} + C, \quad a > 0 \qquad \int \sqrt{a-x^2}\,dx = \frac{1}{2}x\sqrt{a-x^2} + \frac{a}{2}\arcsin\frac{x}{\sqrt{a}} + C, \quad a > 0$$

$$\int \frac{1}{\sqrt{a+x^2}}\,dx = \ln|x + \sqrt{x^2+a}| + C, \quad a \neq 0 \qquad \int \sqrt{a+x^2}\,dx = \frac{1}{2}\left(x\sqrt{a+x^2} + a\ln|x+\sqrt{x^2+a}|\right) + C$$

Maclaurinutvecklingar

$$\begin{aligned}
e^x &= \sum_{k=0}^{\infty} \frac{x^k}{k!} &&= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \\
\sin x &= \sum_{k=1}^{\infty} (-1)^{k-1} \frac{x^{2k-1}}{(2k-1)!} &&= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \\
\cos x &= \sum_{k=0}^{\infty} (-1)^{k} \frac{x^{2k}}{(2k)!} &&= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \\
(1+x)^\alpha &= \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k &&= 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} + \dots \qquad |x| < 1, \quad \binom{\alpha}{k} = \frac{\alpha(\alpha-1)\dots(\alpha-k+1)}{k(k-1)\dots 1} \\
\ln(1+x) &= \sum_{k=1}^{\infty} (-1)^{k-1} \frac{x^k}{k} &&= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \qquad -1 < x \leq 1 \\
\arctan x &= \sum_{k=1}^{\infty} (-1)^{k-1} \frac{x^{2k-1}}{2k-1} &&= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \qquad |x| \leq 1
\end{aligned}$$

Fourierserier

Jämn funktion $f(x) = f(-x)$
Udda funktion $f(x) = -f(-x)$

Fourierserien av en $2L$-periodisk funktion $f(x)$ ges av

$$\frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}$$

där **Fourierkoefficienterna** ges av

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx \qquad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

Den **komplex Fourierserien** av en $2L$-periodisk funktion $f(x)$ ges av

$$\sum_{n=-\infty}^{\infty} c_n e^{in\pi x/L}$$

där de **komplexa Fourierkoefficienterna** ges av

$$c_n = \frac{1}{2L}\int_{-L}^{L} f(x)e^{-in\pi x/L}\,dx$$

Sinusserien av $f(x)$ definierad på intervallet $x \in [0, L]$ ges av

$$\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}\,, \qquad b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

Cosinusserien av $f(x)$ definierad på intervallet $x \in [0, L]$ ges av

$$\frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}\,, \qquad a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

Parsevals identitet för en $2L$-periodisk funktion $f(x)$

$$\frac{1}{L}\int_{-L}^{L} |f(x)|^2\,dx = \frac{1}{2}|a_0|^2 + \sum_{n=1}^{\infty} |a_n|^2 + |b_n|^2$$