(3.1) Let C be given by $x=g(t)$, $y=h(t)$ and $z=l(t)$, $a\le t\le b$. Then $-C$ is given by

$x=g(a+b-t)=\tilde{g}(t)$ $\quad y=h(a+b-t)=\tilde{h}(t)$ $\quad z=l(a+b-t)=\tilde{l}(t)$, $a\le t\le b$.

Then

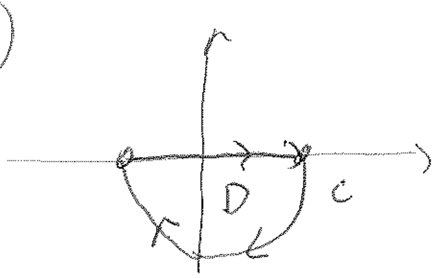
$$\int_{-C}\vec{F}\cdot d\vec{r}=\int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt=\int_a^b\left(P\cdot\tilde{g}'(t)+Q\tilde{h}'(t)+R\,\tilde{l}'(t)\right)dt$$

$$\overset{0.4}{=}\int_a^b\left(Pg'(a+b-t)\cdot(-1)+Qh'(a+b-t)(-1)+R\,l'(a+b-t)(-1)\right)dt=\left|\begin{matrix}a+b-t=s\\-dt=ds\end{matrix}\right|$$

$$\overset{0.2}{=}+\int_b^a\left(Pg'(s)+Qh'(s)+Rl'(s)\right)ds=-\int_a^b\left(Pg'+Qh'+Rl'\right)ds \qquad \Sigma=1$$

$$=-\int_C\vec{F}\cdot d\vec{r}. \quad 0.2$$

(3.2)



Note: C has negative orientation and $-C$ has positive orientation.

$\vec{F}=x\,\vec{i}+(x^3+3xy^2)\,\vec{j}=P\vec{i}+Q\vec{j}$

$$\int_C\vec{F}\cdot d\vec{r}=-\int_{-C}\vec{F}\,d\vec{r}\overset{0.4}{=}-\iint_D\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dA=-\iint_D 3x^2+3y^2-0\,dA$$

$$=\left|\begin{matrix}x=r\cos\theta\\y=r\sin\theta\\0\le r\le 1\\\pi\le\theta\le 2\pi\end{matrix}\right|\overset{0.4}{=}-\int_\pi^{2\pi}\int_0^1 3r^3\,dr\,d\theta=-\frac{3\pi}{4}\left(r^4\right)\Big|_0^1=-\frac{3\pi}{4}. \quad 0.2$$

$\Sigma=1$

(3.3) $\vec{F}=x(x^2+y^2+z^2)^{-p/2}\,\vec{i}+y(x^2+y^2+z^2)^{-p/2}\,\vec{j}+z(x^2+y^2+z^2)^{-p/2}\,\vec{k}$

$$\operatorname{div}\vec{F}=(x^2+y^2+z^2)^{-p/2}-x\cdot\frac{p}{2}(x^2+y^2+z^2)^{-p/2-1}2x+(x^2+y^2+z^2)^{-p/2}-py^2(x^2+y^2+z^2)^{-p/2-1}$$

$$+(x^2+y^2+z^2)^{-p/2}-pz^2(x^2+y^2+z^2)^{-p/2-1}$$

$$=\frac{-p(x^2+y^2+z^2)}{(x^2+y^2+z^2)^{\frac{p+2}{2}}}+\frac{3}{(x^2+y^2+z^2)^{p/2}}=\frac{3-p}{(x^2+y^2+z^2)^{p/2}}\overset{0.8}{=}\frac{3-p}{|\vec{r}|^p}$$

$=0 \Leftrightarrow p=3.$ $\quad 0.2 \qquad \Sigma=1$

(3.4) $\vec{F}$ is conservative iff $\operatorname{curl} \vec{F} = 0$.

$$\operatorname{curl} \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y - z^2 \sin x & x & 2z\cos x \end{vmatrix} = \vec{i}(0-0) - \vec{j}(-2z\sin x + 2z\sin x) + (1-1)\vec{k} = \vec{0}.$$

0.5

Thus, $\vec{F}$ is conservative. Let f be a function such that $\nabla f = \vec{F}$:

$$f_x = y - z^2 \sin x \qquad f_y = x \qquad f_z = 2z\cos x$$

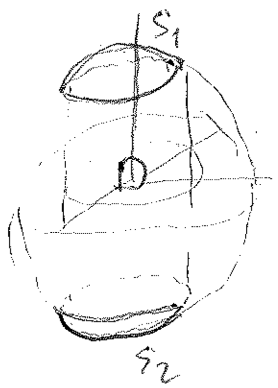
$\Rightarrow f(x,y,z) = y\cdot x + z^2 \cos x + g(y,z)$

$\Rightarrow f_y(x,y,z) = x + g_y(y,z) = x \Rightarrow g_y(y,z) = 0 \Rightarrow g(y,z) = h(z)$

$\Rightarrow f_z(x,y,z) = 2z\cdot\cos x + h'(z) = 2z\cos x \Rightarrow h'(z) = 0 \quad h(z) = C.$

$\Rightarrow f(x,y,z) = y\cdot x + z^2\cos x + C$ 0.5

(3.5)



By symmetry $A(S) = 2A(S_1)$

S_1: $(x,y) \in D$: $0 \le z \le \sqrt{2-x^2-y^2} = g(x,y)$

$D: \{x^2+y^2 \le 1\}$. $\frac{\partial g}{\partial x} = -x\,(2-x^2-y^2)^{-1/2}$ $\frac{\partial g}{\partial y} = -y\,(2-x^2-y^2)^{-1/2}$

$$A(S_1) \overset{0.2}{=} \iint_D |r_x \times r_y|\, dA = \iint_D \sqrt{1 + x^2(2-x^2-y^2)^{-1} + y^2(2-x^2-y^2)^{-1}}\, dA$$

$$= \left| \begin{matrix} x = r\cos\alpha \\ y = r\sin\alpha \end{matrix} \right| = \int_0^{2\pi}\int_0^1 \sqrt{1 + r^2\sin^2\alpha\,(2-r^2)^{-1} + r^2\cos^2\alpha\,(2-r^2)^{-1}} \cdot r\,dr\,d\alpha \quad 0.2$$

$$= \int_0^{2\pi}\int_0^1 \sqrt{1+(2-r^2)^{-1}\cdot r^2}\; r\,dr = 2\pi\int_0^1 \sqrt{\frac{2-r^2+r^2}{2-r^2}}\; r\,dr \qquad Z = 1$$

$$= 2\pi \int_0^1 \sqrt{2}\cdot(2-r^2)^{-1/2}\, r\,dr \; (0.2) = 2\pi\cdot\sqrt{2}\left(-\frac{2}{2}\right)\left[(2-r^2)^{1/2}\right]_{r=0}^{1}$$

$$= 2\pi\sqrt{2}\left(2^{1/2}-1\right) = 2\pi(2-\sqrt{2}) \quad 0.2$$

$A(S) = 2A(S_1) = 4\pi(2-\sqrt{2})$. 0.2