

Problem set 1.

Problem 1.1. Write down the weak formulation of the following initial-boundary value problem:

$$\begin{cases} D_t u(x, t) - D_x((1+x^2)D_x u(x, t)) = t(1-x^2) & \text{in } (-1, 1), t > 0, \\ u(-1, t) = 1, \quad u'(1, t) + 2u(1, t) = 5, & t > 0, \\ u(x, 0) = e^x. \end{cases}$$

Problem 1.2. Solve the following boundary value problem by integrating twice:

$$\begin{cases} -D((2-x)Du) = x^2 + x & \text{in } (0, 1), \\ u'(0) = 1, \quad u'(1) + 3u(1) = 2. \end{cases}$$

Problem 1.3. Find the volume of the solid that lies above the rectangle $R = [1, 3] \times [1, 2]$ and below the surface $z = \frac{1}{1+x+y}$.

Problem 1.4. Read Example 5 in Section 15.3 from the book (Stewart). Then, following a similar approach, evaluate the integral

$$\int_0^1 \int_x^1 e^{x/y} dy dx.$$

Problem 1.5. Evaluate

$$\iiint_E \cos y \, dV,$$

where E lies below the plane $z = x$ and above the triangular region with vertices $(0, 0, 0)$, $(\pi, 0, 0)$ and $(0, \pi, 0)$.