Problem set 2.

Problem 2.1. Find the volume above the rectangle enclosed by the lines $x - y = 0$, $x - y = 1$, $x + y = 0$, and $x + y = 2$ and under the surface $f(x, y) = (x + y)e^{x^2-y^2}$. Hint: use an appropriate change of variables when calculating the double integral associated with the problem.

Problem 2.2. Use the transformation $x = u^2$, $y = v^2$, $z = w^2$ to find the volume of the region bounded by the surface $\sqrt{x} + \sqrt{y} + \sqrt{z} = 1$ and the coordinate planes.

Problem 2.3. Let C is a smooth curve given by $x = g(t)$, $y = h(t)$, $a \le t \le b$ and let $-C$ be given by $x = g(a + b - t)$, $y(t) = h(a + b - t)$, $a \le t \le b$. Let f be a continuous function on C. Show the following equalities.

(a)
$$\int_{-C} f(x, y)\,\mathrm{d}x = -\int_{C} f(x, y)\,\mathrm{d}x;$$

(b)
$$\int_{-C} f(x, y)\,\mathrm{d}y = -\int_{C} f(x, y)\,\mathrm{d}y;$$

(c)
$$\int_{-C} f(x, y)\,\mathrm{d}s = \int_{C} f(x, y)\,\mathrm{d}s.$$

Hint: Use Formulas 3 and 7 from Section 16.2.

Problem 2.4. The base of a circular fence with radius 5 m is given by

$$x = 5\cos t, \quad y = 5\sin t, \quad 0 \le t \le 2\pi.$$

The height of the fence at position (x, y) is given my the function $h(x, y) = 3 + 0.04(x^2 - y^2)$. Suppose that 1 L of paint covers 50 m^2. Determine how much paint you will need to paint both sides of the fence.

Problem 2.5. The force field exerted by an electric charge at the origin on a charged particle at a point (x, y, z) with position vector $\mathbf{r} = \langle x, y, z\rangle$ is $\mathbf{F}(\mathbf{r}) = K\frac{\mathbf{r}}{|\mathbf{r}|^3}$, where K is a constant. Find the work done when the particle moves along a straight line from $(0, 2, 0)$ to $(1, 2, 4)$.