Problem set 3.

Problem 3.1. Let C is a smooth curve given by $x = g(t)$, $y = h(t)$, $z = l(t)$, $a \le t \le b$, and let $-C$ be given by $x = g(a+b-t)$, $y = h(a+b-t)$, $z = l(a+b-t)$, $a \le t \le b$. Prove that if $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$ is a continuous vector field on C, then

$$\int\limits_{-C} \mathbf{F}\,\mathrm{d}\mathbf{r} = -\int\limits_{C} \mathbf{F}\,\mathrm{d}\mathbf{r}.$$

Problem 3.2. A particle starts at the point (-1,0) and moves along the x-axis to (1,0), then along to the semicircle $y = -\sqrt{1-x^2}$ to the starting point. Use Green's Theorem (be careful with the orientation) to find the work done on this particle by the force field

$$\mathbf{F}(x, y) = \langle x, x^3 + 3xy^2 \rangle.$$

Problem 3.3. Let $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$. If $\mathbf{F} = \frac{\mathbf{r}}{|\mathbf{r}|^p}$, find $\operatorname{div}\mathbf{F}$. Is there a value for p for which $\operatorname{div}\mathbf{F} = 0$?

Problem 3.4. Determine whether the vector field $\mathbf{F}(x, y, z) = (y - z^2 \sin x)\,\mathbf{i} + x\,\mathbf{j} + 2z\cos x\,\mathbf{k}$ is conservative. If yes, then find the general form of the potential f of $\mathbf{F}$.

Problem 3.5. Find the area of the part of the sphere $x^2 + y^2 + z^2 = 2$ that lies inside the cylinder $x^2 + y^2 = 1$.