## Problem set 4.

**Problem 4.1.** Verify that Stokes' Theorem is true for the vector field $\mathbf{F}(x, y, z) = y^2z\mathbf{i} - 2y\mathbf{j} + x\mathbf{k}$ where $S$ is the part of the paraboloid $x^2 + y^2 = z$ that lies below the plane $z = 4$ and oriented downward.

**Problem 4.2.** Suppose that $\mathbf{v}(x, y, z) = (z^2 + y^2 + \sin(x^2))\mathbf{i} + (2xy + z)\mathbf{j} + (xz + 2yz)\mathbf{k}$ represents the velocity field of a fluid. Find the circulation of $\mathbf{v}$ around the simple closed curve $C$ (oriented counterclockwise when viewed from above) that is the intersection of the elliptic cylinder

$$(x - 1)^2 + 4y^2 = 16$$

and the plane $2x + y + z = 3$. Hint: Use Stoke's Theorem and a suitable change of variables when calculating the double integral.

**Problem 4.3.** Verify that the Divergence Theorem is true for the vector field

$$\mathbf{F}(x, y, z) = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$$

where $E$ is the ball with radius $a$ centred at the origin.

**Problem 4.4.** Let $C$ be a piecewise smooth, simple closed curve given by

$$\mathbf{r}(\mathbf{t}) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}, \quad d \leq t \leq e,$$

in $\mathbf{R}^3$ which lies in a plane with unit normal vector $\mathbf{n} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ and has positive orientation inherited from that plane. Show that the plane area enclosed by $C$ is

$$\frac{1}{2}\int_C (bz - cy)\,\mathrm{d}x + (cx - az)\,\mathrm{d}y + (ay - bx)\,\mathrm{d}z.$$