Problem set 5.

Problem 5.1.

(a) Write down the boundary value problem for the 2D stationary heat equation on the triangle vertices with $P_1 = (0,0)$, $P_2 = (1,0)$, $P_3 = (1,1)$, with heat conductivity equals $1 + x + y$, constant heat source density equals 1, and constant ambient temperature equals 5 on all sides. On the boundary $x = 1$ the heat transfer coefficient equals 1, on the boundary $y = 0$ the heat transfer coefficient equals 2, while on the rest of the boundary it equals 0. There is no prescribed heat influx at the boundary.

(b) Write down the weak formulation of the problem. Pay particular attention to the fact that the heat transfer coefficient is different on the three different boundary edge segments!

(c) Write down the finite element basis functions ϕ_1, ϕ_2, ϕ_3 for a triangulation that consists of a single triangle $T = \Omega$ with nodes $P_1 = (0,0)$, $P_2 = (1,0)$, $P_3 = (1,1)$.

(d) Compute the elements of the stiffness matrix

$$a_{ij} = a_{ji} = \iint_{\Omega} \lambda \nabla\phi_i \cdot \nabla\phi_j \,\mathrm{d}A + \int_{\Gamma} \kappa \phi_i \phi_j \,\mathrm{d}s.$$

Hint: Pay particular attention to the fact that the heat transfer coefficient is different on the three different boundary edge segments!

(e) Compute the elements of the mass matrix

$$m_{ij} = m_{ji} = \iint_{\Omega} \phi_i \phi_j \,\mathrm{d}A.$$

(f) Compute the elements of the load vector

$$b_j = \iint_{\Omega} f \phi_j \,\mathrm{d}A + \int_{\Gamma} (g + \kappa u_A)\phi_j \,\mathrm{d}s.$$

Hint: Pay particular attention to the fact that the heat transfer coefficient is different on the three different boundary edge segments!

(g) If the vector

$$\mathcal{U} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix},$$

contains the nodal values of the finite element solution of the boundary value problem, write down the linear system of equations that needs to be solved to determine $\mathcal{U}$.