MVE515 Computational Mathematics - Exam

Date: Saturday, 28 October
Time: 8.30-12.30
Room: SB3

- Telephone contact during the exam: Milo Viviani, ext. 5325.
- The final exam is worth 50 points. 20 points are required for a pass grade 3, 30 points for grade 4, and 40 points for grade 5. The bonus points gathered during the course is added to your exam total.
- Aid: you may use the supplied formula sheet.
- Solutions will be announced after the end of the exam on the course homepage.
- You may view the exams on Wednesday, November 22, 10.30-12.30 in MV:L15.

Problem 1.

(a) Write down the weak formulation of the following boundary value problem:

$$\begin{cases} -\mathrm{D}((2x+1)\,\mathrm{D}u) = 4x+1, \quad x \in (0,1), \\ -u'(0) + 2u(0) = 1, \quad u(1) = 0. \end{cases}$$

(5p)

(b) Solve the boundary value problem from (a) by integrating twice. (5p)

Problem 2. Determine whether the vector field

$$\mathbf{F}(x,y,z) = y^2 z^3\,\mathbf{i} + 2xyz^3\,\mathbf{j} + 3xy^2 z^2\,\mathbf{k},$$

is conservative. If yes, then find the general form of the potential f of $\mathbf{F}$. (7p)

Problem 3. Verify that Stoke's Theorem holds for the vector field

$$\mathbf{F}(x,y,z) = -y\,\mathbf{i} + x\,\mathbf{j} - 2\,\mathbf{k},$$

where S is the cone $z^2 = x^2 + y^2$, $0 \le z \le 4$ oriented downward. (8p)

Problem 4. Verify that the Divergence Theorem holds for the vector field

$$\mathbf{F}(x,y,z) = \langle x^2, xy, z \rangle$$

and the solid E bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane. (10p)

Please turn over!

Problem 5.

(a) Write down the boundary value problem for the 2D stationary heat equation on the triangle with vertices with $P_1 = (1,0)$, $P_2 = (1,1)$, $P_3 = (0,1)$, where the heat conductivity is equal to $2+x+y$, the heat source density is equal to $1+xy$, and the ambient temperature is equal to constant 2 on all sides. The heat transfer coefficient is equal to $y+x-1$ at the boundary. There is no prescribed heat influx at the boundary. (5p)

(b) Write down the weak formulation of the problem. (5p)

(c) Write down the finite element basis functions ϕ_1, ϕ_2, ϕ_3 for a triangulation that consists of a single triangle $T = \Omega$ with nodes $P_1 = (1,0)$, $P_2 = (1,1)$, $P_3 = (0,1)$. (3p)

(d) Compute the element a_{13} of the stiffness matrix

$$a_{ij} = a_{ji} = \iint\limits_{\Omega} \lambda \nabla\phi_i \cdot \nabla\phi_j \, \mathrm{d}A + \int\limits_{\Gamma} \kappa\phi_i\phi_j \, \mathrm{d}s.$$

(2p)