MVE515 Formula sheet

- $\int_C f\,\mathrm{d}s = \int_a^b f(\mathbf{r}(t))\sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\,\mathrm{d}t$
- $\int_C \mathbf{F}\cdot\,\mathrm{d}\mathbf{r} = \int_a^b (\mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\,\mathrm{d}t = \int_a^b P(\mathbf{r}(t))x'(t) + Q(\mathbf{r}(t))y'(t) + R(\mathbf{r}(t))z'(t))\,\mathrm{d}t$
- $\iint_S f\,\mathrm{d}S = \iint_D f(\mathbf{r}(u,v))|\mathbf{r}_u(u,v)\times\mathbf{r}_v(u,v)|\,\mathrm{d}A$
- $\iint_S \mathbf{F}\cdot\,\mathrm{d}\mathbf{S} = \iint_D \mathbf{F}(\mathbf{r}(u,v))\cdot(\mathbf{r}_u(u,v)\times\mathbf{r}_v(u,v))\,\mathrm{d}A$ or $\iint_S \mathbf{F}\cdot\,\mathrm{d}\mathbf{S} = \iint_D \mathbf{F}(\mathbf{r}(u,v))\cdot(\mathbf{r}_v(u,v)\times\mathbf{r}_u(u,v))\,\mathrm{d}A$, depending on the orientation of S.
- If a surface S is the graph of a function $z = g(x,y)$; that is, S is given by the parametric equations $x = x$, $y = y$ and $z = g(x,y)$, then the upward normal (non-normalized) is
$$\mathbf{r}_x(x,y)\times\mathbf{r}_y(x,y) = -\frac{\partial g}{\partial x}\mathbf{i} - \frac{\partial g}{\partial y}\mathbf{j} + \mathbf{k}$$
- Green's Theorem: $\int_C \mathbf{F}\cdot\,\mathrm{d}\mathbf{r} = \iint_D (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})\,\mathrm{d}A$
- Stokes Theorem: $\int_C \mathbf{F}\cdot\,\mathrm{d}\mathbf{r} = \iint_S \operatorname{curl}\mathbf{F}\cdot\mathrm{d}\mathbf{S}$
- Divergence Theorem $\iint_S \mathbf{F}\cdot\,\mathrm{d}\mathbf{S} = \iiint_E \operatorname{div}\mathbf{F}\,\mathrm{d}V$
- Integration by parts in 2D: $\iint_\Omega \phi\nabla\cdot\mathbf{F}\,\mathrm{d}A = \int_\Gamma \phi\mathbf{F}\cdot\mathbf{n}\,\mathrm{d}s - \iint_\Omega \mathbf{F}\cdot\nabla\phi\,\mathrm{d}A$
- Integration by parts in 3D: $\iiint_\Omega \phi\nabla\cdot\mathbf{F}\,\mathrm{d}V = \iint_\Gamma \phi\mathbf{F}\cdot\mathbf{n}\,\mathrm{d}S - \iiint_\Omega \mathbf{F}\cdot\nabla\phi\,\mathrm{d}V$
- Change of variables: $\iiint f(x,y,z)\,\mathrm{d}V = \iiint f(x(u,v,w),y(u,v,w),z(u,v,w))\left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right|\,\mathrm{d}V$
- Polar coordinates: $x = r\cos\theta$, $y = r\sin\theta$, $\frac{\partial(x,y)}{\partial(r,\theta)} = r$
- Spherical coordinates: $x = r\sin\phi\cos\theta$, $y = r\sin\phi\sin\theta$, $z = r\cos\phi$,
$$\frac{\partial(x,y,z)}{\partial(r,\phi,\theta)} = r^2\sin\phi$$
- For a sphere centered at the origin, with radius a written in spherical coordinates $\mathbf{r}(\phi,\theta) = a\sin\phi\cos\theta\,\mathbf{i} + a\sin\phi\sin\theta\,\mathbf{j} + a\cos\phi\,\mathbf{k}$:
$$\mathbf{r}_\phi\times\mathbf{r}_\theta = a\sin\phi\,\mathbf{r}(\phi,\theta)$$
$$|\mathbf{r}_\phi\times\mathbf{r}_\theta| = a^2\sin\phi.$$