MVE515 Computational Mathematics - Re-exam

Date: Wednesday, 20 December
Time: 14.00-18.00

- Telephone contact during the exam: Robert Forslund, ext 5325.
- The final exam is worth 50 points. 20 points are required for a pass grade 3, 30 points for grade 4, and 40 points for grade 5. The bonus points gathered during the course is added to your exam total.
- Aid: you may use the supplied formula sheet.

Problem 1. Write down the weak formulation of the following boundary value problem:

$$\begin{cases} -(\cos(x)u'(x))' = 1 + x^2 \quad x \in (0,1), \\ -u'(0) = 1, \quad \cos(1)u'(1) + u(1) = 1 \end{cases}$$

(7p)

Problem 2. Evaluate

$$\iiint_E e^y \, \mathrm{d}V,$$

where E lies below the plane $z = x$ and above the triangular region with vertices $(0,0,0)$, $(1,0,0)$ and $(0,1,0)$. (7p)

Problem 3. Let $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$, $|r| = \sqrt{x^2+y^2+z^2}$ and $\mathbf{F}(x,y,z) = \frac{\mathbf{r}}{|\mathbf{r}|^3}$. Find its general potential function. (5p)

Problem 4. Find the surface area of the part of the sphere $x^2+y^2+z^2 = 4$ that lies outside the cylinder $x^2+y^2 = 1$. (7p)

Problem 5. Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$ where $\mathbf{F}(x,y,z) = yz\,\mathbf{i} + 2xz\,\mathbf{j} + e^{xy}\,\mathbf{k}$ and C is the circle $x^2+y^2 = 16$, $z = 5$, oriented counter clockwise when viewed from above. (6p)

Problem 6. Let $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$ be a vector field on $\mathbb{R}^3$ with continuously differentiable components and let f be a continuously differentiable scalar field on $\mathbb{R}^3$. Show that

$$\mathrm{div}\,(f\mathbf{F}) = f \mathrm{div}\,\mathbf{F} + \mathbf{F} \cdot \nabla f.$$

(5p)

Please turn over!

Problem 7. Write down the weak formulation of the following boundary value problem in 3D.

$$\begin{cases} -\nabla \cdot (a\nabla u) + cu = f & \text{in } \Omega, \\ u = u_{\mathrm{A}} & \text{on } \Gamma_1, \\ a\mathrm{D}_N u = g & \text{on } \Gamma_2, \end{cases}$$

where a, c, f, g are functions, u_{A} is a constant and $\Gamma = \Gamma_1 \cup \Gamma_2$ (Γ_1 and Γ_2 are non-overlapping). (8p)

Problem 8.

(a) Write down the finite element basis functions ϕ_1, ϕ_2, ϕ_3 for a triangulation that consists of a single triangle $T = \Omega$ with nodes $P_1 = (0, 0)$, $P_2 = (-1, 1)$, $P_3 = (0, 1)$. (3p)

(b) Compute the element m_{12} of the mass matrix

$$m_{ij} = m_{ji} = \iint_{\Omega} \phi_i \phi_j \, \mathrm{d}A.$$

(2p)